# The Reorganization of Mathematics for the Emergency

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WITH MATHEMATICS heading the list of requirements for almost every function connected with the emergency, there are few who question the significance of that field. The potential contributions of mathematics are clearly recognized, and immediate results demanded. The responsibility for seeing the problem as a whole, and defining the present role of mathematics in the educational program, falls directly on those interested in the teaching of mathematics.

The situation presents certain problems to the teacher of mathematics. In place of a rather clearcut set of objectives, as defined in college entrance requirements or preparation for specific vocations, we have a miscellaneous assortment of preparations to make, and mathematical abilities to develop. There are the requirements, for example, of the air force, military and naval programs, industry, and the home front. Some of these are being voiced emphatically, if somewhat vaguely. Others have little dramatic appeal, are little heard from, and are in danger of being neglected. It is our problem to analyze and clarify this heterogeneous assortment of purposes, verify them, and translate them into a program of action in the form of a

mathematics curriculum. It is the purpose of this paper to outline the problems of such a survey.

Intelligent planning of any school activity must include consideration of the needs of society, as well as of the characteristics of the individual pupil. In normal times the social requirement was primarily the definition of good citizenship. Today we must also recognize certain emergency demands, particularly the mobilization of manpower, and the maintenance of the home front. While it is true that we must never lose sight of the individual pupil, with his particular needs and destiny, it becomes clear, as we consider the problems of manpower and the home front, that we think in terms of three major groups of pupils, as determined by their future roles:

1. Those who will enter the armed forces, including the military, naval, and air.
2. Those who will be engaged in industrial production, including those with aptitudes that justify special training for the fields of science.
3. Those who will have their major responsibilities, as consumers, to maintain the home front.

It is worth while briefly to investigate what

we know about the special needs of each of these groups in order to define precisely the effective role of mathematics. While reliable information about any of them is still inadequate, we may note the important sources where it can be sought, what data are available, and what remains to be secured.

## MATHEMATICAL REQUIREMENTS OF THE ARMED SERVICES

Information as to the needs of those about to enter the armed forces comes from two general sources: recommendations of individuals and committees that have studied these requirements; and the nature of the training programs set up in the service branches themselves.

Among the former we have statements by such persons as General Kuter,[1] Admiral Nimitz,[2] and Lieutenant Paul C. Smith.[3] All such statements stress emphatically the need for mathematical competence, and deplore the present lack of it, among those entering the various branches of service. They should clarify any lingering doubt as to the need for mathematical literacy in the personnel of the modern armed forces.

The Progress Report[4] of the Subcommittee on Education for Service, of the War Preparedness Committee, is the result of a careful and extensive study of the mathematical requirements of military service, as well as of certain non-military

1. BRIGADIER GENERAL L. S. KUTER, address to Institute on Education and the War, reported in *Education for Victory*, 1: 19, October 1, 1942.
2. Letter from ADMIRAL W. C. NIMITZ to LOUIS J. BUDVOLD, published in THE MATHEMATICS TEACHER, ■: 88, February 1942.
3. LIEUTENANT PAUL C. SMITH, "The Navy and the Schools," THE MATHEMATICS TEACHER, 35: 248–252, October 1942.
4. W. L. HART, "Progress Report of the Subcommittee on Education for Service of the American Mathematical Society and the Mathematics Association of America," THE MATHEMATICS TEACHER, 34: 279–304, November 1941.

defense activities. The Subcommittee makes the very useful suggestion that, since we cannot predict the branch of service in which a given pupil may be called to serve, the mathematical program should be adequate for the service with the highest mathematical requirements. They suggest the desirability of college preparatory mathematics through trigonometry and solid geometry, together with additional work in applications of arithmetic.

While useful in general, such blanket requirements are not sufficiently precise for classroom teachers, or those concerned with curriculum construction. Superficially, the recommendation of a year of geometry, for example, appears quite definite. Actually, it still leaves many questions. What geometry shall we teach? Is it geometry for the purpose of developing critical thinking, the nature of proof, ability to analyze propaganda, and so on? Or is the nature of proof to be incidental to the study of geometric relationships, constructions, or applications in technical, industrial, or scientific problems? Attempts to do both in the same course have failed in the past.

More precise definition of the outcomes to be expected are needed not only in geometry, but in algebra, solid geometry, and trigonometry as well. Such information can come only from a detailed study of the requirements of each of the service branches. Such study as has been made of the texts and classroom activities of the military and naval preflight training courses, while needing further amplification, is sufficient to show that *we must think in terms of mathematical processes and applications, rather than in blanket course recommendations.* Ability to solve problems in navigation involving *air speed* and *drift* by means of vector parallelograms or triangles, for example, is found to be important in both air services. Since there is not sufficient time in the training school for these concepts to be developed from the beginning, such problems must be in-

troduced in the high school. Some classes in geometry are now dealing with this problem as an application of parallelogram construction.

Until an organized effort has carried through an adequate study of the service requirements of mathematics, the classroom teacher must depend on the technical manuals of the army,[5] the publications of the Civil Aeronautics Authority,[6] and the Air Age Education Series,[7] together with periodicals and textbooks in the field of aeronautics. All these, of course, *are suggestive mainly for applications of what is being taught rather than for reorganizing the course itself.* As our information becomes more complete we shall be able to match the topics in our courses against the mathematical abilities needed in all the services, including, of course, the service organizations of women. We should then be able to specify precisely *what* algebra, geometry, trigonometry, and elementary mechanics we must include in the curriculum.

## MATHEMATICS FOR INDUSTRY

The indignation and dismay of a public suddenly made aware of its mathematical illiteracy when confronted with the technical requirements of industry is graphically portrayed in a recent paper by Hedrick,[8] who at the same time points out some excellent implications for curricular reorganization. The necessity for women to replace men in industrial positions whenever possible recalls the favorite question of the past generation: "Why should girls study mathematics?" It is to be regretted that we did not have the answer then as we have it now.

The role mathematics teaching must play in contributing to the effective manpower is well illustrated in the need to provide industrial workers, both men and women, competent to fill technical positions that require mathematical background. Our contribution must be thought of in terms both of individual efficiency and of numbers. Our procedure must be such as to make certain that no one who has the ability to fill a given role will fail to achieve through any mathematical bottleneck. In terms of interest—whom have we been losing? Can any of those who have been dropping out be retained? In terms of guidance, what is the potential role of the individual pupil in the national effort? How can he be brought to the required level of efficiency?

These questions point clearly to the need for overcoming the traditional prejudice against girls in mathematical and scientific curricula; for discovery and retaining pupils of both sexes with special aptitudes for mathematics and science; and for interesting and securing achievement in the large groups of pupils with a minimum of mathematical aptitude who, in the past, have avoided mathematics.

These are new problems. There is little in our previous experience to guide us. The most profitable suggestions come from the schools set up by industrial plants themselves. Kadushin,[9] for example, reports that, at Lockheed, relatively few employees were willing to enroll in classes in mathematics. For the majority it was necessary to bring in the mathematics instruction incidentally in courses in blueprint reading, tool design, sheet metal layouts, and the like. No mention was

5. War Department, *Technical Manual, Mathematics for Pilot Trainees*, TM 1-900, Washington, April 1942.
   War Department, *Technical Manual, Air Navigation*, TM 1-205, Washington, November 25, 1940.
6. U. S. Department of Commerce, CAA, *Aerodynamics for Pilots*, Bulletin 26, Washington, 1938.
   U. S. Department of Commerce, CAA, *Practical Air Navigation*, Bulletin 24, Washington, 1938.
7. GEORGE OSTEYEE, *Mathematics in Aviation*, Macmillan, New York, 1942.
8. E. R. HEDRICK, "Mathematics in the National Emergency," THE MATHEMATICS TEACHER, 35: 253–59, October 1942.

9. J. KADUSHIN, "Mathematics in Present-day Industry," THE MATHEMATICS TEACHER, 35: 260–64, October 1942.

made of mathematics as such. This experience bears out the warning of Douglass,[10] made some years ago, that for the majority of pupils classroom procedure in mathematics have been calculated to destroy interest in mathematics rather than to create it.

The classroom procedures described by Kadushin as being effective in his experiences are based on the fact that "learning goes on best in the degree that the individual sees and feels the significance to his own felt needs of what he does."[11] He found that mathematical processes are introduced most effectively through real applications that are clearly useful, and easy to grasp. It is interesting, in the connection, to note that Wheeler arrives at the same conclusion through the principles of the psychology of learning: "No transfer will occur unless the material is learned in connection with the field to which transfer is desired. Isolated subjects and ideas do not integrate."[12]

Evidently the techniques that Kadushin found successful are those that are basic in good teaching—to proceed from significant, understandable, problem situations, to the abstract mathematical principle, rather than the reverse. The immediate problem is to discover real applications of mathematics in industrial and technical situations, which the pupils will recognize as important. Again we need an organized research for such materials. Until this is available, they must come from the numerous texts, some of them excellent, on shop mathematics, from the cooperation of the school shop men, and from semitechnical periodicals. When it becomes generally recognized that women have an important place to fill in industry, it is

probable that materials of this sort will be as interesting to the girls as to the boys.

### The Home Front

The primary social requirement with respect to the groups we have been considering has been mobilization of manpower, in terms both of individual efficiency and numerical adequacy. The drastic changes in our way of life that total war demands, however, throws the spotlight on the hitherto neglected and undramatized responsibilities of the consumer. The fact that we have left his problems to the last is typical. It should not imply, however, that the needs of all other groups may be met before his are considered. It is well to recall that the collapse of the home front was the immediate cause of the defeat of Germany in the last war. It is pleasant to think that "it can't happen here," but at the same time we should take steps to assure that it does not.

There has always been lacking, in our economic system, an educated, enlightened, socially responsible consumer who knows what goods and services he wants, and how to get them; whose earning, wisely spent, will finance adequately the activities of industry; whose tastes and preferences among its products will direct its productive efforts as they should be directed, to worthwhile commodities; and whose investments will cause expansion in the directions determined by social needs. Today this requirement still exists, with the additional demand that while he does so, the consumer finance the most expensive war in history.

Among the understandings needed by the consumer in the present emergency are those that arise from increasing incomes, coupled with the decline in opportunity for spending money. Rationing, substitutions, price ceilings, and whatever other efforts are adopted to avoid the natural consequences will be effective only to the degree that there is general understanding of their purpose. These problems

10. HARL R. DOUGLASS, "Let's Face the Facts," THE MATHEMATICS TEACHER, 30: 56–62, February 1937.
11. J. KADUSHIN, op. cit., p. 262.
12. R. H. WHEELER, "The New Psychology of Learning," Tenth Yearbook, National Council of Teachers of Mathematics, Columbia University, New York, 1935.

are clearly quantitative in their nature. We recognize mathematics as the medium for thinking, communicating, and problem solving in the realm of quantitative ideas. We recognize, also, that the tendency to discontinue the study of arithmetic beyond the eighth grade has contributed materially to the mathematical illiteracy of our adult population. Why not bring these two thoughts together, in a study of the materials made available by the Consumer Division of the OPA,[13] the Department of Agriculture,[14] current newspapers and periodicals, and the community itself?

## PROPOSALS FOR ACTION

It is clear that our analysis up to this point has revealed many problems and little useful information to guide our attacks on them. Yet the situation demands action, in the light of our best judgment, and on the basis of what information is available. If we take action with inadequate data, we shall make mistakes. Yet the greatest mistake of all would be to take no action. If provision is made for interchange of information, we can capitalize on successes, avoid repetition of errors, and discover in even greater detail the nature of needed information. It will be useful to summarize here the directions in which we need to move.

1. Special provision must be made for those about to leave school to enter industry or the armed forces. It is unfortunately true, that we have fallen heir to the consequences of a movement away from mathematics. Many of our juniors and seniors, both boys and girls, have had no secondary school mathematics, and are now recognizing the need for it. We have a responsibility to this group: first to meet, as adequately as possible, their immediate mathematical needs; and second, to awaken an interest in mathematics that will lead them to seek further study in the field, rather than to avoid it.

Neither of these purposes can be served by the traditional "good, stiff, review of the fundamentals." Such reviews may yield superficial results as measured by tests on the operations, but achieve nothing whatever to alleviate the current criticism that our graduates who can perform the operations with creditable speed and accuracy do not know how to apply these processes in a problem situation. Such reviews do have the positive effect of killing whatever interest in mathematics the pupil brings to the course. Classroom procedures directed to this end should incorporate the best practices in diagnostic testing and remedial teaching.

What is clearly indicated is a study of real applications of the processes the pupils are most likely to use. Such a course is outlined in the report on a "Wartime Program of Mathematics and Physics,"[15] for the National Association of Secondary School Principals. The applications should be sought in the situations in which the graduates will probably find themselves. Experience in a few schools, at least, has demonstrated the effectiveness of such a course, dealing with problems of geometric constructions, measurement, graphs, formulas, and applications of arithmetic. We need further experimentation, by imaginative and well-informed teachers, to discover what can be done, and the effective ways of doing it.

2. The mathematics curriculum as a whole should be appraised in the light of the purposes it has to serve. The present organizational form (as outlined in the Fifteenth Yearbook of the National Coun-

13. U. S. Office of the OPA, Consumer Division. *Brief Bibliography for Consumers in Wartime.* Washington, OPA, Consumer Division, July 1942.
   U. S. Office of the OPA, Consumer Division. *Economics of the Home Front.* Washington, OPA, Consumer Division, July 1942.
14. U. S. Department of Agriculture, Bureau of Home Economics and Consumer Counsel Division. *Be a Victory Pioneer In Your Home.* Washington, 1942.

15. Bulletin of the National Association of Secondary School Principals, *Secondary Education and the War*, pp. 40–51, October 1942.

cil, for example) should remain as the basis of revisions. The enthusiastic suggestion that we "throw the curriculum out the window and start over" has little to recommend it beyond its enthusiasm. The fact that we have misused our mathematical fields, and mistaken their purposes in the curriculum, suggest that we search for more effective use of what we have, while exploring for new possibilities.

The key questions in this revision should be these: How and by whom, are these mathematical processes used? What are the real applications that make them significant? How do I expect the pupil to use them? If the results of these questions are negative for any process, then the problem is defined for further study—the activities of industry, the armed forces and the home front, have a way of moving on ahead, and we may have to catch up. If further study fails to reveal an important functional purpose, either directly or indirectly as a prerequisite, then the process must make way for those that are functional.

With regard to the applications that we discover, our question is, where do they fit in? We have seen that the vector triangle and parallelogram are important. When should the solution of such problems be considered? Ability to understand and interpret maps constructed from various projections appears to be important, not only in the armed forces, but on the home front. Does this appear as a mathematical responsibility? If so, where does it fit into the curriculum in mathematics? Are there other uses of quantitative thinking that we should be dealing with, that have been neglected through over-emphasis on logical sequence in mathematics?

In reorganizing our curriculum, we should feel free to move topics from one year to another so far as we can do so without violating the necessities of sequence. It may appear, for example, that only a few pupils will find time to take a course in trigonometry, although an understanding of the trigonometric functions is widely required. In that case, the right triangle, and the trigonometric functions, should be given extensive treatment in geometry, where they will be available for the majority of pupils.

3. We must make a definite attack on the problem of developing mathematical literacy in the groups with low mathematical aptitude. Our failure with this group in the past has been due, in part at least, to the abstract nature of the principles we have been emphasizing. The operations have to often been divorced from the significant problem situations in which they occur. In terms of the present-day literature,[16] the *meaningful* aspect of mathematics, in its logical characteristics, has been basic in our teaching. *Significance*, which is the importance of the process as recognized by the pupil, has been largely ignored, or considered as "enrichment."

Kadushin, Hedrick, and Wheeler, as we have seen, stress the importance of significance in the materials dealt with in the course. Present-day psychology of learning sets it up as a fundamental necessity. True, it had been possible, in the past, to place the emphasis on the meaningful nature of the field. We have been successful in developing principles, and processes, logically, in a systematic way. Applications have been used largely as practice material for drilling on the principle. Such success as we have experienced was due to the characteristics of the group we were dealing with. Through superior intelligence, they could often restore the abstract principle to its functional setting, and realize its significance. This we called "transfer of training." For the new group we can depend on no such "soft pedagogy." The principles must be restored to their original setting—we must, in other

16. National Council of Teachers of Mathematics, *Sixteenth Yearbook, Arithmetic in General Education*, p. 158, New York, Bureau of Publications, Columbia University, 1941.

words, deal with applications from the start.

Only when the pupil realizes the importance of the process, and the nature of its applications, is he ready to study its logical and systematics place in the field of mathematics. This meaningful aspect of the process must then be carefully developed, in order that he may become independent and effective in applying it to new situations to which it is appropriate.

Both meaning and significance are concepts that must be basic in revising the curriculum. It would be as disastrous to "go overboard" on significance in the future as it has been to over-emphasize meaning in the past. Because mathematics is a logical, systematic field, highly sequential in its development, we must consider the vertical organization of topics, to be certain that the prerequisite for each has been mastered before it is introduced.

4. It is obvious that these proposals contemplate not only immediate action, but a long-time program of reorganization as well. The research and experimentation involved in this reorganization will require provisions for centralization of efforts, and for cooperative study. There is need for continuously evaluating of the recommendations issued by individuals and committees to keep abreast of curricular revisions and recommend feasible practices, studying of current publications, and discovering applications that occur in various fields. Clearly, some such central clearing house is essential in the experimental attack we have been considering. For this purpose, the teacher-education institutions should assume a major responsibility in the work that lies ahead, in cooperation with such groups as the Subcommittee on Mathematics of the California Committee on Education, and the National Council of Teachers of Mathematics.

Besides this "service" function, the teacher-education institutions have other important obligations. There is, for example, the need for in-service cooperation with teachers. This is becoming especially important for the teacher being "converted" from other fields to teach mathematics. At last reports[17] there were shortages of mathematics teachers in about one-forth of the states, and surpluses of teachers of English, history and the social studies in about the same number. The inevitable administrative readjustments are obvious. Teachers moved into mathematics from other fields must find immediate and effective assistance from the teacher-education institutions. If this can be done, such teachers, far from being a liability, may prove to have a contribution in a new point of view with regard to the significance of our field.

Probably the most effective single contribution of the teacher-education institutions would be the establishment of summer workshops for the study of curriculum and instruction in mathematics. Teachers and administrators who are engaged in an active program of curriculum construction would find it useful to bring in their projects for continued study with opportunities for consultation and guidance. The materials that would be made available in such workshops, together with the opportunity for interchange of ideas among those actively engaged in curriculum revisions, should provide stimulating conditions for effective work. At the same time such an organization could provide consultants from fields other than education—military and naval services, the air force, industry, and various activities of the home front—with which the mathematics curriculum is concerned.

## Conclusion

In his charming collection of stories entitled "Beside the Bonnie Briar Bush," Ian MacLaren portrays the personalities of a people that he clearly loves for their limitations as well as for their strengths.

17. Reported by Office of Education in *Education for Victory*, 1: 9, October 1942.

He reveals as an outstanding characteristic of these people their economy of words. In one of his scenes an inhabitant recently come to the valley is exclaiming over a sunset: "If that isn't glorious, tell me what is?"

"Man," he is gently reprimanded by one more accustomed to the ways of the valley, "Ye must e'en save a word for John 3: 16."

In keeping with this spirit, no downpour of rain deserved anything better than the description of "a wee bit misty." There was need to save a word for the deluge.

One might wish that we had been as frugal with our vocabulary in education. For the past twenty or thirty years we have been expending, in a reckless way, words like *challenge*, *crisis*, *opportunity*, and *responsibility*. We should have saved some of them for the present situation.

Today we see mathematics once more, for all practical purposes, a general school requirement—a state of affairs few of us expected to see. We see interest in mathematics, and in the welfare of mathematics teaching, at a higher level than at any time in our educational history. Industry, business, and the armed forces are ready to cooperate in making our field significant.

It would, perhaps, in this situation, be quite natural for us to assume, in a smug way, that events had proved we were right all along; that the content and methods of the past are justified; that the procedures we had been using were to be used again, only a little harder this time. On this point we can do no better than to quote Hedrick:[18]

When Mathematics was a general school requirement, not only in the graded schools but also in the secondary schools, and even in the colleges and universities, we sinned in the face of every decent precept. We earned the reputation which mathematics itself does not deserve, namely that our work was disciplinary, rather than for any usefulness.

To repeat this mistake in the face of present needs will be permanently to discredit our field.

We may recognize, on the other hand, that we have a problem, involving the difficulties of research into the significant applications of our field, while at the same time overcoming the natural aversion of our pupils to precise thinking. We can perceive that the adequate solution of this problem will be the permanent establishment of mathematical literacy as an outcome to be achieved in the schools of the future.

Yet if I should say that we stand at the crossroads, your reaction would be: "Are we back there again?"

The answer to this is the same as that to the plea "back to the 3 R's"—we have never been there.

18. E. R. HEDRICK, *op. cit.*, p. 257.

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### Official Notice
### The National Council of Teachers of Mathematics

# A Proposal for Mathematics Education in the Secondary Schools of the United States

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## IMPORTANCE OF MATHEMATICS

THE FACT that mathematics is so important is not surprising to one who is properly informed as to the contributions which it has made to the other great fields of knowledge, but many people, including some of our general educators, are still unaware of the strategic place which mathematics really occupies in world affairs today. It should be the business of those of us who are interested primarily in the subject to help to make clear just where and how mathematics can be of real service to the other great branches of learning and what can be done to secure these services by a better teaching of mathematics in the schools. The war emergency has brought out the great importance of mathematics, but even a war is not enough to convince some people as to the value of mathematics in the education of American citizens.

## THE UNIVERSALITY OF MATHEMATICS

In this complex civilization which we are now entering, a knowledge of mathematics is becoming increasingly important.[1] This does not mean that everyone should be trained to be a mathematician, but it does mean that every well educated citizen in America should know a reasonable amount of mathematics and also that he should be trained to use it in an intelligent manner. To put it in another way, some of our people will act in the role of producers of mathematics while others for the most part will be merely consumers. It should be our concern to see that these consumers are taught to appreciate the work of the producers just as some people who may not be able to produce grand opera may nevertheless appreciate a good opera when they hear it.

If many of the present-day ills of this country could be traced to their origin, the cause would be found to be a lack of knowledge of the mathematics underlying the situation or a failure to appreciate its important implications. Much of our failure to understand some of the problems we have to meet in everyday life is due primarily to a lack of knowledge and appreciation of the simplest kinds of arithmetic. This explains why it has been so easy for unscrupulous salesmen to hoodwink so many housewives and also why it is so difficult to get people to investigate the ultimate cost of certain types of buying on the installment plan. It also explains why people have lost their life savings because of a lack of ability in or appreciation of some of the most obvious types of informational arithmetic[2] and the moral values that are involved.

As Mr. Garvan,[3] of the Chemical Foundation, once pointed out. "The whole discussion is arithmetic." Why is it that so many of our graduates of the secondary school and the colleges today know so little of budgeting, the cost of unemployment, the benefits of foreign trade, the cost of crime and the resulting punishment of the offender, and the cost of taking care of the needy, the insane, and the homeless? One large factor here is the neglect of informational arithmetic all

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[1] Reeve, W. D., "The Universality of Mathematics, THE MATHEMATICS TEACHER, February, 1930.

[2] Judd, C. H., "Informational Versus Computational Mathematics," THE MATHEMATICS TEACHER.

[3] Garvan, Francis P., A Speech before the Friends of the Catholic University of America, p. 10.

along the line. Too much time has been spent on computational arithmetic, and often of a very low order at that, in so far as the real needs of society are concerned. This situation should be remedied immediately.

### FUTURE NEEDS

To make mathematics education function at its highest in this country we need to evaluate anew not only the materials of instruction, but also to improve our methods of teaching and learning mathematics. The improvement of content makes necessary a reshaping of aims and the development of a new nation-wide cooperative program. The improvement of teaching and learning will involve the raising of standards for teachers entering the profession and improved opportunities for the further education of teachers in service.

Mathematics has been adapted to mass education. The natural result is a lowering of scholarship, a leveling down instead of a leveling up, but an increase in influence which on the whole will result in good because more people will have a chance without necessarily jeopardizing our opportunity to discover the pupils of genius.

### THE NEED FOR MORE SCHOLARLY TEACHERS

The fact that our country is new and that the high school population has increased so fast in the last thirty years (It is now close to 7,200,000) has made it impossible for us to demand as high qualification for teachers as we should like to have had.

Another significant factor is the fact that our unprecedented industrial and economic growth has resulted in men leaving the teaching profession to go into other lines of work. Due to the war, the exodus of male teachers is not only large, but alarming. Even women are leaving the teaching profession to take up some sort of war work. While we may admit the natural superiority of women with children, we must not fail to realize that for

many of our women teaching is not a permanent profession. What is the mental effect? I think it has led to a lack of interest in the proper preparation and background for teaching mathematics in the schools. As a result we do not have among either men or women teachers enough people of scholarly minds. The mathematical background of many of our teachers is decidedly limited, to say nothing of their deficiencies along other lines.

Many people are teaching mathematics in the secondary schools today who have not had any mathematics courses of collegiate grade. The situation is even worse than that in some places. When a superintendent of schools insists on putting a teacher, who has had no mathematical training, in charge of high-school classes in mathematics because he has no other use for him, the situation is unsatisfactory. Even now when the shortage of teachers is so acute, the policy of delegating people, who are not prepared, to teach mathematics may be worse than no teacher at all, unless of course they seek immediate preparation. The National Council of Teachers of Mathematics should begin to study such practices and make plans to improve them. We could do this if we had sufficient group consciousness, group enthusiasms, group loyalty, the courage of our convictions and some financial help.

### SUGGESTIONS FOR IMPROVEMENT

If we are to make an attempt to improve the mathematical situation in this country, the following suggestions should be helpful:

1. Two types of scholarly men and women should be developed: First, research scholars whose major interest is to expand mathematics vertically: second, teacher scholars who are primarily interested in the horizontal expansion of the subject.

2. We need courses in methods and in professionalized subject matter for students who intend to be college teachers of

mathematics. The teaching of college mathematics in many places is far below the standard.[4]

3. There is no question that salaries of teachers in this country should be raised so as to attract capable men and women who will remain in the teaching profession. When that is done, we shall have to raise our academic requirements as well if we are to bring about genuine reform. I recently read about a teacher who resigned a position paying $70 per month to take a position in government service at $70 per week.

Already the increasing number of college graduates with good academic background in mathematics who are enrolling in our teacher training institutions in America augurs well for the future. When the war is over we shall have an over-supply of candidates for teaching positions. We should then be able to raise our standards and employ only the best of all those who are available.

4. It seems to me that we might require some training in science, at least in physics, of all prospective teachers of mathematics. In some schools in America the work in mathematics is already being correlated in this way.

5. We must cooperate with subject matter specialists in other fields in demanding more recognition on the programs at such general educational meetings as state associations, sectional meetings, and teachers institutes. Some state associations are so completely under the control of secretaries (often men of meager academic training), that it is almost impossible for a subject matter group to secure any outside talent without themselves providing the necessary funds. As Pierce[5] puts it "Convention speakers rarely present a topic which has even

remote value to the teacher's problems in the classroom. Most of them advance their pet theories, discuss their research work, or talk about economic or political problems with which they are none too familiar. Most teachers go to Conventions to receive instruction on the nature of the child and his proper nurture. High sounding phrases, patriotic oratory, or research dissertations hold little of interest and less of value to the average teacher.

6. As fast as we improve the teachers of mathematics we can lift the level of the articles in THE MATHEMATICS TEACHER and in *The National Council Yearbooks*. However, we shall still face the problem of getting these publications in the hands of teachers where they will be more widely read. While it may be a surprise to some, there are plenty of mathematics teachers who do not even know that there is such an organization as the National Council of Teachers of Mathematics.

It is no exaggeration to say that we should have a membership of at least 10,000 in the National Council of Teachers of Mathematics. We can do better than that if each person now a member secures one new member. This would not be difficult, but our present membership must take greater interest in the cause. If we can publish and distribute free monographs to our present members, this will do a great deal to augment our national growth. Financial difficulties due to lack of general support among mathematics teachers makes this hard to accomplish.

### NEED FOR A FURTHER STUDY OF OUR PROBLEMS

Since the epoch-making report of the National Committee on Mathematical Requirements[6] on "The Reorganization of Mathematics in Secondary Education," in 1923, we have been able to carry on the work started by this committee and to approach the problem of mathematical education from a national point of view

[4] Seidlin, Joseph. *A Critical Study of the Teaching of Elementary Mathematics*. Bureau of Publications, Teachers College, Columbia University, Contributions to Education, No. 482.

[5] Pierce, A. Lester, "Why Teachers Duck Conventions." *The Clearing House*, November, 1942, p. 174.

[6] A report prepared under the auspices of the Mathematical Association of America.

through the work done by the recent Joint Commission of The Mathematical Association of America and the National Council of Teachers of Mathematics on "The Place of Mathematics in Secondary Education."[7] However, the present war emergency makes it necessary for us to consider possible and necessary reorganization of the course of study for secondary schools to fit pupils to help win the war.

### A Proposal

The following proposal represents a worthwhile and necessary procedure and one we would be able to carry on if financial support were forthcoming. It relates to mathematics education in the elementary school (grades one to six inclusive), the junior high school (grades seven to nine inclusive), the senior high school (grades ten to twelve inclusive), the junior college, the senior college, teachers colleges, and the field of adult education.

As previously implied, any attempt to improve mathematics education in the above *schools* will involve an elaborate program of educating the necessary teachers and also of educating the public to whom we must look for continued support of a new curriculum.

The proposal will be divided into two parts, the first part relating to immediate needs and plans and the second part to forward-looking possibilities if financial support can be obtained.

### Immediate Needs and Plans

1. Now, if ever, is the time to develop nation-wide cooperation in mathematics covering the schools previously mentioned. It should not be controlled by any extra mural examination boards and should contain what the leaders who cooperate think should be the mathematics of the future. It seems absurd that we

[7] See the report of the Joint Commission which was published as the Fifteenth Yearbook of the National Council of Teachers of Mathematics. Bureau of Publications, Teachers College, 525 W. 120th St., New York, N. Y. Price $1.75 postpaid for a bound volume.

should spend the energy and money, as at present in this country, in order to produce all of the various state and city syllabi with the result that they fail utterly to meet the needs of the hour. The results of such a cooperative effort should be accompanied by a teachers manual, the purpose of which would be to give helpful suggestions to the teacher as to what should be done in the classroom.

### Suggested Omissions and Additions

In arithmetic we should omit the following:

(1) A very large part of the work in common fractions as of no practical use any longer and as discouraging, meaningless, and unnecessarily difficult for children.

(2) About one-half of the work with decimals, including addition and subtraction of "ragged decimals," and the useless cases in multiplication and division as still persist.

(3) Such parts of work in denominate numbers as are no longer used, certainly not more than two denominations. Some advance is already being made in this direction.

(4) Square and cube roots except by the use of tables. The theory belongs in algebra, if anywhere.

(5) Such extended computations as have no applications within the child's range of knowledge. This includes difficult and discouraging work in the mechanics of multiplication and division, which operations are now practically done by calculating machines when met with in actuarial work or in physics and astronomy.

This suggestion of elimination does not mean the lessening of of time allowed to the subject. On the contrary it means the replacing of the useless material by that which belongs to the twentieth century instead of the seventeenth. The nature of this new material will possibly require

even more time, although this will probably be unnecessary for the average pupil who does not stand in need of the arithmetic of special fields of commerce, industry, and other technical lines. This does not mean that material should be omitted solely because of its age, but age alone should not settle the question.

In arithmetic we should add the following:

(1) The story of our numerals, not to learn but to enjoy; not extensive but as related to the growth of knowledge.

(2) Number games as a reward, not as a task.

(3) Applications to school and home life, with problems actually brought in by the class.

(4) Similarly, applications to the social and civic life of the class. These are easily used to advantage as soon as the elimination of useless material allows time for the real problems of home life (cooking, cost of gas or electricity, cost of heat, and the like), games, and what in general may be called "the arithmetic of environment." Considerable progress has been made in this field, but much more can be made if we save time as suggested.

(5) The use of simple algebra such as that from Grade VII on, as an aid to the solution of problems of the kind mentioned.

(6) Informal geometry[8] as a part of arithmetic from Grade VII on.

(7) Mechanical computation if and when the school shall acquire simple types such as the future will certainly demand.

Besides these additions of an elementary nature there are the very important fields of accounting, technical training, national economy, and science. We have been faced

by problems of tremendous importance to this country, of which one is that of isolation. How high can we raise the tariff walls, how completely can we live within our borders, importing little and exporting in the same proportion? There is also the problem of taxation—one that touches every home in America. These and others like them are essentially mathematical questions for our Junior High Schools, Senior High Schools, and Junior Colleges and they are well within the grasp of students in such departments of education. What are we doing to answer them? Practically nothing for the average pupil. It is from such important storehouses that we can and should find material to replace what is now obsolete in arithmetic and algebra. But too many of our present teachers see these needs only in spots; they are blind to the problem as a whole. One teacher may do something with the economical question of buying on the installment plan, and another may venture upon the question of tariff, while a third may spend time on budgets or some useless labor on useless projects, but a new body of teachers must be trained to see the great importance of economics in all such fields.

All this is concerned with the blending of arithmetic and elementary algebra. The failure to realize that there is no definite boundary between the two is also a relic of two centuries ago.

In algebra we should omit the following:

(1) Most of the present work in the operations with polynomials, such as have no application in social life, in ordinary business, and in science. These are relics of two or three centuries of wasted energy.

(2) Most of the mere puzzle and uninteresting "applied" equations, the time being given to work mentioned below.

(3) Elaborate cases of simultaneous equations, especially such trick solutions as have no applications in science or industry.

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[8] See page 79 of the Report of the Joint Commission (Fifteenth Yearbook) as to the nature of this work.

In algebra we should add the following:

(1) A much greater use of the equation in the manipulating of formulas actually needed in connection with
  (a) Physical problems, the meaning and significance of which are within the grasp of the pupils.
  (b) Commercial problems.
  (c) Problems relating to social activities—graphs, investments, support of government, budgets, household interests.

(2) Series as related to pupils' interests—population, compound interest, physical problems, simple computations by logarithms (leading to the explanation of the slide rule), and other social needs.

(3) A more purposeful use of the idea of function, of dependence of one variable upon another. In particular, such dependence as relates to business, to science, and to the problems of daily life.

(4) A more intimate relation of physical problems to algebra as has long been cultivated in European schools and as is suggested in No. 1 above. The lack of this recognition is one of the greatest weaknesses in the teaching of algebra in America. To overcome it requires a radical departure from the present training of teachers of mathematics. Our teachers of the entire subject must know more of science and of the applications of mathematics to the life of the twentieth century. Opposed to this will be many general educators who look upon the matter solely from the standpoint of the theory of teaching. We must start *de novo* in the training of teachers of mathematics if we hope to succeed.

In geometry it will be necessary to omit:

(1) About two-thirds of the traditional propositions to be proved fully. The real purpose of logical geometry can better be secured by retaining only the necessary basal propositions, introducing more original matter, and reducing the deduction aspects of the course for many pupils.

(2) All attempt at remembering definitions and statements of propositions, thus replacing mechanical methods by real thinking. In particular, abandon all attempts to define the indefinable and minimize the ultra-logical beginning of geometry.

In geometry we should add:

(1) A modern beginning, establishing the truth of the propositions informally. This movement is already under way.

(2) A more carefully planned use of models made by the pupils so as to have geometry seem more real.

(3) A large number of applications to science. Here we have hardly made a beginning.

(4) A number of abstract exercises, applying logic to new situations. Here we have made a fair beginning.

(5) The idea of generalizing propositions, especially in the study of varied shapes of the standard figures. This will introduce the negative line, angle, and area, opening up a field of interest new to the pupils. For this we need much better-trained teachers, not only in the theory of education but also in the modern field of geometry.

(6) A large number of applications to the science of Air and Marine Navigation. [9]

[9] See Bradley, A. D., *Mathematics of Air and Marine Navigation*, American Book Co., New York, 1942. Price, $1.00. See also Osteyee, George, *Mathematics in Aviation*, Air-Age Education Series. Macmillan, New York, 1942, and The 17th Yearbook of The National Council of Teachers of Mathematics, *A Source Book of Mathematical Applications*. The Bureau of Publications, Teachers College, 525 W. 120th St., New York, N. Y. Price, $2.00 postpaid for a bound volume.

To take another view of the question of geometry, consider its relation to the larger problem of life and of the universe about us. Upon this David Eugene Smith has written at some length in his *Poetry and Mathematics* in which he has dwelt upon Geometry and Religion—not sectarianism and the minor disputes of New England in the remote past, but the larger question of world religion and of the nature of the space about us and the evidence of a world to come. All this can be treated apart from "creeds and rites" as *obiter dicta* in our opening of the door to real mathematics; but, again, it can be accomplished only by one who has given serious attention to its vast possibilities; in other words, to the teacher who has escaped from the emptiness of many courses in education.

The Church has come to capitalize the discoveries of science and the rigid thinking of mathematics, instead of fearing them, and has been enormously strengthened thereby. Whatever contest there ever was between Science and Religion was not carried on by the great leaders on either side, but by those of lesser intelligence. This truth should be a part of the Geometry which teachers of the subject should imbide in their educational training, but which, under present conditions, is never brought to their attention.

In Trigonometry we should, except for pupils who show unusual ability, omit:

(1) Those parts of the subject rarely used except in the highest computations of astronomy. I refer to a considerable part of the work in trigonometric equations and to the formulas for half-angles except where real applications are available.

(2) Trigonometric formulas in general except as needed in ordinary work in physics and elementary astronomy.

In Trigonometry we should also recall the fact that the subject is essentially a part of algebra and has relatively little relation to theoretical geometry. Here, again, we must begin *de novo* the training of teachers. In particular:

(1) The whole subject of the imaginery number rests chiefly on trigonometry. This can be easily introduced in the high school, but we need a new generation of teachers to realize its importance. The whole theory of vector analysis starts here, and although not suitable for the high school (except for the parallelogram of forces) it is easily taught to college freshmen.

(2) The elements of trigonometry are much easier than the relatively useless material which should be dropped from highschool algebra.

In College Algebra there should be dropped (always excepting the highly-gifted mathematician) all that cannot be used later in mathematics or in elementary science. This should be replaced by work in analytic geometry or the calculus.

Analytic Geometry is today taught about as it was in the eighteenth century—with no apparent purpose to arouse and maintain interest. Its application to algebra, elementary geometry, physics, and astronomy is generally neglected.

There is here a rich field for the training of an entirely new brand of teachers, with new interests and with new vision of the beauties and the utilities of the subject.

The Calculus should abandon its present introduction as written by mathematicians for mathematicians. As it stands at present, it is too often taught as it was a century ago.

To begin with a rigid, ultra-logical presentation of the subject is to discourage almost all students. It is like the old way of teaching a foreign language by devoting the first few months to ultra-technical grammar.

We should begin with simple applications, apply intuition and solve many problems. After all this, we should give the rigid theory to such as can absorb it.

2. The preparation of a permanent mathematics exhibit at Teachers College, Columbia University and other similar institutions showing the relation between the various parts of mathematics itself and also the correlation of mathematics with the other great fields of knowledge like art, music, and science.

The project at Teachers College is well started, but to be of greatest value, it should be permanently housed and placed so that teachers coming, as they do from all over the country, can study it in detail. This work should be made continuous so that the exhibit can be kept modernized in every way. What we have done here has already stimulated teachers in other parts of the country to start their own exhibits. This shows the influence which we already have upon the country as a whole, but this is clearly only a beginning.

3. For the sake of large numbers of teachers who may not be able to come to New York in person, it would he helpful if we could publish a monograph showing the essential and most striking features of this exhibit and similar ones in other places. It is just possible that this monograph could be definitely related to the cooperative effort and teachers manual referred to in No. 1 above. Naturally, we hope to make this available for distribution at a nominal price.

4. The preparation and publication of important monographs of a mathematical nature such as the following:

a. A monograph on the story of our numerals. This monograph is published and is intended for use as supplementary reading material in the social studies.[10]

b. A similar monograph to that described in (1) above to be used in the same way should be prepared on "The Social Aspects of Arithmetic."

c. A third study already completed will be of interest here.[11] It deals with "Mathematics and the Social Studies" and gives an idea of the kind and amount of mathematics that is needed from the beginning of the junior high school through the junior college for a proper understanding of the books that relate to the present social studies curriculum.

d. Another monograph on "Great Men of Mathematics"—what they did and what effect their contribution has had on mathematics and civilization should be prepared for use not only in mathematics, but also in the social studies. Why should not our citizens be made aware of the important contributions to civilization of such men as Pythagoras, Descartes, and Newton?

e. Several important and helpful monographs should be prepared by getting together under one cover some of the most significant articles that have been written in such fields as Arithmetic, Algebra, Geometry, and Trigonometry. A wisely selected list of such monographs made available to teachers at a nominal cost would improve the teaching of mathematics in the next generation.

5. Financial support is greatly needed for standing committees of the National Council of Teachers of Mathematics such as

a. Geometry Committee. This committee has a report published in monograph form. The Committee should be continued.

b. Policy Committee. It is the function

[10] Smith, David Eugene and Ginsberg, Jekuthial. *Numbers and Numerals*. Bureau of Publications, Teachers College, Columbia University, 525 W. 120th St., New York, N. Y. Price, 25¢ postpaid.

[11] Helmich, Eugene W., *The Mathematics in Certain Elementary Social Studies in Secondary Schools and Colleges*. Bureau of Publications, Teachers College, Columbia University, Contributions to Education, 706.

of this committee to outline matters of policy for the approval of the National Council and in a general way to carry on the work previously done by the National Committee on Mathematical Requirements and the Joint Commission previously referred to.

c. A Committee on the Problem of teaching of Mathematics to Gifted Children. The most retarded pupil in the American school system today is the gifted pupil of scholarly mind. In a democracy, if anywhere, the training of our future leaders is imperative and today we are certainly not very successful in the task.

d. A Committee on the Problem of Teaching Pupils of Low Ability in Mathematics. Along with the necessity of training leaders in a democracy comes the need for intelligent followership. Our pupils of low ability are not learning much mathematics or anything else that is of permanent worth, and when they fail they spend too long a time in doing it. Moreover, such pupils are unhappy all through school and go out into life slaves to every political demagogue who comes along and promises to make their lives richer by overthrowing what our fathers have built up at a great sacrifice.

With financial support we can carry on certain important duties which should ameliorate the present situation and point the way toward a better day.

6. Financial support is urgently needed right now for a Commission of the Mathematical Association of America on "The Utilization of Advanced Students of Mathematics." It is the purpose of this Commission to see what can be done to place in strategic teaching positions the trained graduates of our advanced schools. The trouble now is that the committee does not have funds to pay the necessary expenses of getting together.

Until recently we had an oversupply of teachers of mathematics, but we have never had an oversupply of thoroughly well-trained teachers. Neither is it clear that we would have enough teachers of the ordinary type to fill the positions throughout the country if the schools were properly organized for teaching purposes.

7. In order to successfully carry on a nation wide cooporative program, the exhibit, and the monographs, and to coordinate the work involved in this proposal, we should at once have a staff of at least three research scholars who could give all of their time to these projects. It should be their duty to collect and interpret material and to help to get it in the best possible shape for general use. Later this group should doubtless be enlarged.

8. The official journal of the National Council of Teachers of Mathematics, THE MATHEMATICS TEACHER, needs not only to be increased in size, but to be improved in quality. It is impossible under "hand to mouth" policy under which we operate to make large improvements. With a reasonably small subsidy, this publication could soon be made the best in the country, and because of that fact it would attract many teachers who now miss whatever stimulation and help it has to offer.

Instead of having several periodicals struggling along under separate management, and none of them really meeting our needs, we should have one first-class publication to which the best minds in the land would be honored to contribute. Lack of money makes this unrealizable at the present.

9. At great expense and at a small profit (all of the labor of the editor and the contributors is gratuitous), The National Council of Teachers of Mathematics, beginning in 1926, has now published 17 Yearbooks in spite of the great depression. This has been done at great sacrifice in order to keep alive the interest and devotion of the teachers of mathematics to the cause they represent. It is becoming increasingly clear that these Yearbooks,

which in appearance are second to none in the other fields, are considered most important contributions to mathematical education. We feel that the friends of mathematics will wish to see these publications continued. This can be guaranteed by a very small annual expenditure, although the cost of publication is increasing; but the books cannot be made to reach enough teachers without a subsidy.

10. Last, but not least, we need an Appraisal Committee whose duty it should be to conduct a national survey of the needs and possibilities of the various parts of the country as far as mathematics is concerned. This survey would help to give us an adequate picture of the present situation so as to better enable us to know just what steps to take in the building of the future. I consider this a very important project. I would suggest that three of our best leaders in the field should be selected for this work and that they give full time to the study for at least one year.

### Subsequent Needs and Plans

If the first part of this double proposal is granted, we should have ample opportunity to see that this is only a beginning. To realize our ambitions with respect to mathematical education in this country, we must have a permanent mathematical center to which teachers of mathematics, educators, and interested laymen can look for guidance. The second part of this proposal is, therefore, that we establish at some important center as soon as feasible a Mathematical Institute on the secondary level, that is, through the sophomore year in college, which will do for teachers at that level what the Einstein Institute at Princeton does for the advanced students.

This institute should be so organized that the very best possible training in the country for prospective and experienced teachers of mathematics through the junior college could be obtained. It should also be a bureau of service for teachers in the field who might need assistance concerning materials or methods of instruction. Finally it should be the coordinating center for a continuation of the cooperative program so that the course of study in mathematics might be continually improved. In making improvements the interest and cooperation of all members of the profession should be encouraged.

The establishment of laboratory schools in connection with leading state and other teacher training institutions which would cooperate with the institute would furnish the necessary ideas for improvement in methods of teaching.

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### Notice to Subscribers

Have you bought the Seventeenth Yearbook? If not, see the back outside cover of this issue. Professor Harold Fawcett of Ohio State University writes recently "I have just received my copy of the 17th yearbook and hasten to say that in my opinion it will be one of the most useful and helpful yearbooks in the entire series."

### Notice

The National Council of Teachers of Mathematics will hold an all-day regional meeting in Chicago, Illinois on Saturday, March 6, 1943. The program for this meeting will be announced in the February issue of The Mathematics Teacher.

# The Importance of Mathematical Education

## An Old Story, Oft Repeated

*By* WILLIAM L. SCHAAF

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> The study of mathematics is only for those who need it, those who can do it, and those who like it.—Anon.

> To ask whether a child has an aptitude for mathematics is equivalent to asking whether he has an aptitude for reading and writing.—C. A. LAISANT.

SOME half a dozen years ago, referring to the precarious position of mathematical education, Professor E. T. Bell asserted that "in the coming tempest only those things will be left standing that have something of demonstrable social importance to stand on. . . . The harsh attrition has already begun. Are not mathematicians and teachers of mathematics in liberal America today facing the bitterest struggle for their continued existence in the history of our Republic? American mathematics is exactly where, by common social justice, it should be—in harassed retreat, fighting a desperate rear-guard action to ward off annihilation. Until something more substantial than has yet been exhibited, both practical and spiritual, is shown the non-mathematical public as a justification for its continued support of mathematics and mathematicians, both the subject and its cultivators will have only themselves to thank if our immediate successors exterminate both. Taking a realistic view of the facts, anyone but an indurated bigot must admit that mathematics has not yet made out a compelling case for democratic support. . . . This must be done, and immediately, if mathematics is to survive in America."[1]

Doubtless there is all too much truth for complacency in these remarks; on the other hand, it is curious to observe that from time to time similar fears are expressed concerning the disappearance of mathematical studies. For example, the following passage, although written nearly seventy years ago, has a peculiarly contemporary ring to it;[2] the then Bishop of Manchester, England, in a report on education in the United States and Canada, said that he "found a widely spread fear in many directions, of the encroachments of the physical sciences on the general domain of education, as though they were trying to occupy the whole ground. We have already seen the fate of the Classics: and even the Mathematics, I was told, can hardly maintain their position even in the Universities of Yale and Harvard. They get 'crowded out' by other studies of more 'immediate practical utility'." Today it would seem that the *social studies* were doing most of the crowding and pushing, though I venture to add, with probably no greater success, in the long run, than the physical sciences of two generations ago.

The potential virtues of the study of mathematics have been acclaimed and denied many times over. A recent writer[3] on secondary education suggests that "the remarkable and unquestioned value of mathematics to the race has been partly responsible for its misuse as a means of promoting the educative growth of pupils at the secondary school level." Much the same could probably be said with equal force concerning required "freshman mathematics."

Let us turn the hour-glass back a century and listen to the astute and critical Augustus de Morgan:[4]

[1] *American Mathematical Monthly*, Nov. 1935, 42: 558 ff.

[2] Quoted by Isaac Todhunter, in The Conflict of Studies and other Essays, London, Macmillan, 1873, p. 31.

[3] R. O. Billett: Fundamentals of Secondary School Teaching, Boston, Houghton-Mifflin, 1940, p. 311.

[4] Augustus de Morgan, On the Study and Difficulties of Mathematics. London, 1831.

We shall now make a few remarks on the advantages to be derived from the study of mathematics, considered both as a discipline for the mind and a key to the attainment of other sciences. It is admitted by all that a finished or even competent reasoner is not the work of nature alone; the experience of every day makes it evident that education develops faculties which would otherwise never have manifested their existence. It is, therefore, as necessary to *learn to reason* before we can expect to be able to reason, as it is to learn to swim or fence, in order to attain either of those arts. Now, something must be reasoned upon, it matters not much what it is, provided that it can be reasoned upon with certainty. . . . Now the mathematics are peculiarly well adapted for this purpose. . . . These are the principal grounds on which, in our opinion, the utility of mathematical studies may be shown to rest, as a discipline for the reasoning powers. But the habits of mind which these studies have a tendency to form are valuable in the highest degree. The most important of all is the power of concentrating the ideas which a successful study of them increases where it did exist, and creates where it did not. A difficult position, or a new method of passing from one proposition to another, arrests all the attention and forces the united faculties to use their utmost exertions. The habit of mind thus formed soon extends itself to other pursuits, and is beneficially felt in all the business of life.

Elsewhere in the same tract de Morgan presses the argument in slightly different words:

In what then, may it be asked, does the real advantage of mathematical study consist? We repeat again, in the actual certainty which we possess of the truth of the facts on which the whole is based, and the possibility of verifying every result by actual measurement, and not in any superiority which the method of reasoning possesses, since there is but one method of reasoning. . . . In mathematics, therefore, we reason on certainties, on notions to which the name of innate can be applied, if it can be applied to any whatever. Some, on observing that we dignify such simple consequences by the name of reasoning, may be loth to think that this is the process to which they used to attach such ideas of difficulty. There may, perhaps, be many who imagine that reasoning is for the mathematician, the logician, etc., and who, like the Bourgeois Gentilhomme, may be surprised on being told that, well or ill, they have been reasoning all their lives. And yet such is the fact; the commonest actions of our lives are directed by processes exactly identical with those which enable us to pass from one proposition of geometry to another.

And in still another passage he drops the "disciplinary theme" and alludes very briefly to more "direct" values of mathematical study, as follows:

As a key to the attainment of other sciences, the use of the mathematics is too well known to make it necessary that we should dwell on this topic. In fact, there is not in this country any disposition to undervalue them as regards the utility of their applications. But though they are now generally considered as a part, and a necessary one, of a liberal education, the views which are still taken of them as a part of education by a large proportion of the community are still very confined.

It should be remembered, of course, that when de Morgan wrote these lines, non-Euclidean geometry had not yet become generally known;[5] also that "faculty psychology" and implicit faith in "transfer of learning" were widely accepted, tacitly, if not patently.

A generation before the voice of John Perry was raised, the prolific and eminent Todhunter, discussing contemporary education in England, wrote as follows:[6]

Let me now say somthing as to the special advantages of mathematics. Leaving aside such points as are well known and obvious. I should like to draw attention to the inexhaustible variety of the problems and exercises which it furnishes; these may be graduated to precisely the amount of attainment which may be possessed, while yet retaining an interest and value. It seems to me that no other branch of study at all compares with mathematics in this. . . . Another great and special excellence of mathematics is that it demands earnest voluntary exertion. . . . Nor do I know any study which can compete with mathematics in general in furnishing matter for severe and continued thought. . . . I speak now, as on former occasions, of studies as they present themselves to minds of average power and of ordinary conditions.

Todhunter was frank to admit that there were certain disadvantages attached to the study of mathematics, and that it was not a simple discipline. He says, for example:

[5] The earliest publication of Lobachevski's creation was in 1829; Bolyai's *Tentamen* appeared in 1832–33.

[6] Isaac Todhunter: Op. cit.

The study of mathematics certainly requires steady perserverance in combating with difficulties; but the processes employed and the results to which they lead are both so important and so interesting, that it is not surprising that the pursuit is found eminently inviting.

Just about at the turn of the century, however, the entire question of mental discipline began to be severely frowned upon, if not completely jettisoned. Yet scarcely a few years later (1912), with greater frankness than some of his colleagues did, Schultze emphatically declared that "mathematics is primarily taught on account of the mental training that it affords, and only secondarily on account of the knowledge of facts that it imparts. The true end of mathematical teaching is power and not knowledge." Meanwhile the influence of several reform movements in mathematical education was slowly becoming manifest,—movements that had begun in 1900 or a little earlier with John Perry in England, Felix Klein in Germany, and E. H. Moore in America. Protesting vigorously against mathematical training that sought to create mathematicians and enable candidates to pass examinations, Perry contended that methods of instruction then in vogue might well destroy reasoning power and create dislike for mathematics. In the interest of the average citizen he pleaded urgently for less formalism, free use of "experimental mathematics," and greater liberality with respect to basic assumptions. The Perry movement in England thus foreshadowed by a quarter of a century some of the significant features of general mathematics of the junior high school in America. In Germany, Klein with equal sincerity advocated both a psychological organization of the subject matter of mathematics, as well as a closer relation between algebra and geometry by making the function concept the unifying idea. In his own words:

We, who are called the reformers, would put the function concept at the very center of instruction, because, of all the concepts of mathematics of the past two centuries, this one plays the leading role wherever mathematical thought is used. We would introduce it into instruction as early as possible with constant use of the graphical method, the representation of functional relations in the $X$-$Y$ system, which is used today as a matter of course in every practical application of mathematics. In order to make this innovation possible, we would abolish much of the traditional material of instruction, material which may in itself be interesting, but which is less essential from the standpoint of its significance in connection with modern culture.

Professor Moore, in his presidential address in December 1902 asked for less formalism and greater emphasis on the practical side; for more laboratory mathematics,—"things, not words"; for correlation between arithmetic, algebra and geometry; for free play of the intuition; and for the fusion of mathematics with science.

During the next two or three decades there ensued a lively controversy concerning the value and desirability of substantially universal mathematical education. Shouts and murmurs became clearly audible; claims and counterclaims were extravagently put forth. Not only did mathematicians and teachers become more articulate,—even the "general educators" became increasingly vigorous in their praise or denunciation (chiefly the latter) of mathematics; even men of letters occasionally joined in. Let us see what some of our literary friends have said.

Thus from the pen of the inimitable Stephen Leacock, we find the conviction presumably shared by many a layman:

Take mathematics; how can you shorten the subject? That stern struggle with the multiplication table, for many people not yet ended in victory, how can you make it less? Square root, as obdurate as a hardwood stump in a pasture—nothing but years of effort can extract it. You can't hurry the process.

Or pass from arithmetic to algebra: you can't shoulder your way past quadratic equations, or ripple through the binomial theorem. Indeed, the other way—your feet are impeded in the tangled growth, your pace slackens, you sink and fall somewhere near the binomial theorem with the calculus in sight on the horizon. So died, for each of us, still bravely fighting, our mathematical training: except only for a set of people called "mathematicians"—born so, like crooks. Yet would we leave mathematics out? No, we hold our cross.

Somewhat less subtle and in more serious mood are the following passages from two other writers:

The secret of happiness is curiosity. Now curiosity is not only not roused; it is repressed. You will say there is not time for everything. But how much time is wasted! Mathematics. . . . A mediaeval halo clings around this subject which, as training for the mind, has no more value than whist-playing. I wonder how many excellent public servants have been lost to England because, however accomplished, they lacked the mathematical twist required to pass the standard in this one subject? As a training in intelligence it is harmful: it teaches a person to underestimate the value of evidence based on other modes of ratiocination. It is the poorest form of mental exercise—sheer verification; conjecture and observation are ruled out. A study of Chinese grammar would be far more valuable from the point of view of general education. All mathematics above the standard of the office boy should be a special subject, like dynamics or hydrostatics. They are useless to the ordinary man. If you mention the utility of a mathematician like Isaac Newton, don't forget that it was his pre-eminently anti-mathematical gift of drawing conclusions from analogy that made him what he was. And Euclid—*that* frowsy anachronism! One might as well teach Latin by the system of Donatus. Surely all knowledge is valueless save as a guide to conduct?—NORMAN DOUGLAS: *South Wind.*

I earned my living at school slavery, teaching to children the most useless, the most disastrous, the most soul-cramping branch of knowledge wherewith pedagogues in their insensate folly have crippled the minds and blasted the lives of thousands of their fellow creatures—elementary mathematics. There is no more reason for any human being on God's earth to be acquainted with the binomial theorem or the solution of triangles, unless he is a professional scientist,—when he can begin to specialize in mathematics at the same age as the lawyer begins to specialize in law or the surgeon in anatomy,—than for him to be expert in Choctaw, the Cabala, or the Book of Mormon. I look back with feelings of shame and degradation to the days when, for a crust of bread, I prostituted my intelligence to wasting the precious hours of impressionable childhood, which could have been filled with so many beautiful and meaningful things, over this utterly futile and inhuman subject. It trains the mind,—it teaches boys to think, they say. It doesn't. In reality it is a cut-and-dried subject, easy to fit into a school curriculum. Its sacrosanctity saves educationalists an enormous amount of trouble, and its chief use is to enable mindless young men from the universities to make a dishonest living by teaching it to others, who in their turn may teach it to a future generation.—WILLIAM J. LOCKE: *The Morals of Marcus Ordeyne.*

Naturally, among the many voices heard, those of mathematics teachers and mathematicians have been raised most often. Generally theirs has been an honest, sincere defense of the subject. Not always, however: here and there a couageous spirit has frankly condemned a false assumption or an ineffectual procedure. But why not let them speak for themselves? With the reader's kind indulgence, therefore, we shall once more set forth, in random fashion save for a slight gesture as to chronological sequence, a number of passages culled from the writings of those who, for one reason or another, have earned the right to be heard. We leave it to the reader to weigh them and then decide for himself.

But widespread as are the applications of mathematics and enormous as is its practical value, it may be justly urged that to the large majority of people its importance, though great, is *indirect*, and that the average citizen has but little need of mathematical facts, or even opportunity to use them beyond the merest elements of arithmetic.—J. W. A. YOUNG (1906).

The subtlety, delicacy and accuracy of mathematical processes have the highest educational value, both direct and indirect. To treat them as mechanical routine, not susceptible of explanation or illumination from a higher point of view, is to destroy in large measure the value of mathematics as an educational instrument, and to aid in arresting the mental development of the pupil.—NICHOLAS MURRAY BUTLER (1911).

It would be an error to infer from the great usefulness of mathematics to our civilization, an equal practical usefulness to every individual. . . . If mathematics, however, had no value as a mental discipline, its teaching in the secondary schools could hardly be justified solely on the grounds of its bread-and-butter value.—ARTHUR SCHULTZE (1912).

Still more dangerous is the plea that every educated man should have some idea of a subject [geometry] of such wide utility. Apart from the claims of many other branches of knowledge, it has a further demerit in that the object of teach-

ing the subject is implied to be the acquisition of encyclopaedic knowledge, rather than the development of the mental faculties. The old conception of education as the acquisition of information is dead, and it least becomes mathematicians to do anything to revive it. The use of justifications of this type, even though it be only in secondary positions, is likely to defeat the aims of those who advance them and to do much harm to educational ideals.—G. St. L. Carson: *Essays on Mathematical Education* (1913).

And I would ask you to remember one thing more. The whole world is going through a transformation, due in part to scientific and mechanical invention and in part to the growth of separate nations, each with its own methods and ideals, of which no man can see the outcome. Our function, the function of all teachers, is to produce men and women competent to appreciate these changes and to take their part in guiding them so far as may be possible. Mathematical thought is one fundamental equipment for this purpose, but mathematical teaching has not hitherto been devoted to it, because the need has but recently arisen. But now that it has arisen and is appreciated, we must meet it or sink, and sink deservedly. Neither the arid formalism of older days nor—I say it in no spirit of disrespect—the workshop reckoning introduced of late will save us. The only hope lies in grasping that inner spirit of mathematics which has in recent years simplified and co-ordinated the whole structure of mathematical thought, and in relating this spirit to the complex entities and laws of modern civilization.—G. St. L. Carson: *Essays on Mathematical Education* (1913).

The burden of proof rests upon those who wish to displace mathematics as a required subject for high school pupils. The statement that recent investigations have thrown doubt on the disciplinary value of mathematical study is absolutely without justification. The chief disciplinary value of mathematics is in the training of the reasoning powers it affords. The writer has gone over the literature on transfer of training quite recently and does not know of any experiments that involved training in logical reasoning. It is absurd to contend that experiments based on the marking of certain letters on a printed page or guessing the size of pieces of paper, will enable one to draw valid conclusions with regard to the training afforded by the study of mathematics.—Charles N. Moore: *American Mathematical Monthly*, Feb. 1916.

Everyone knows that mathematics should have a tremendous influence upon the training of the citizen, and yet we so often use our time and energies solely in puttering as to the statis-

tical results of teaching subtraction by this little method or that. While giving all such matters due consideration, can we not take the larger view? Can not the general educator bring his experience to help us in the special fields to make better citizens? Can not we who love mathematics and believe in its larger possibilities bring all our skill to help the general educator develop the great elements of life in the souls intrusted to our care?—David Eugene Smith: "Mathematics in the Training for Citizenship," *T. C. Record*, May 1917.

Nobody can be a good reasoner unless by constant practice he has realized the importance of getting hold of the big ideas and of hanging on to them like grim death. For this sort of training geometry is better than algebra. The field of thought in algebra is rather obscure, whereas space is an obvious insistent thing evident to all. Then the process of simplification, or abstraction, by which irrelevant properties of matter, such as color, taste, and weight are put aside is an education in itself. Again the definitions and propositions assumed without proof illustrate the necessity of forming clear notions of the fundamental facts of the subject matter and of the relations between them. . . . The learner is not confronted with any symbolism which bothers the memory by its rules. Also, from the very beginning the reasoning, if properly guided, is dominated by well-marked ideas which guide each stage of development. Accordingly, the essence of logical method receives immediate exemplification.—A. N. Whitehead.

Every great study is not only an end in itself, but also a means of creating and sustaining a lofty habit of mind; and this purpose should be kept always in view throughout the teaching and learning of mathematics.—Bertrand Russell.

The primary purpose of the teaching of mathematics should be to develop those powers of understanding and of analyzing relations of quantity and of space which are necessary to an insight into and control over our environment and to an appreciation of the progress of civilization in its various aspects, and to develop those habits of thought and of action which will make those powers effective in the life of the individual.—National Committee Report on the Reorganization of Mathematics (1923).

The purpose, therefore, of requiring our young people to study mathematics is to give them a knowledge of what the science means, and to make it possible for them to continue further in one or more of its branches as their tastes or needs require. Let us again repeat that it is not the purpose to make mathemati-

cians of all of them, nor even to try to make most of them capable of solving a quadratic equation or of proving the theorem of Pythagoras.—DAVID EUGENE SMITH (1925).

The amount of training in algebra needed by the people at large has been a burning issue in educational circles for a long time. Eminent authorities have stated that all the algebra that can be assimilated should be taught to all the people capable of learning; just as great authorities have asserted that the amount actually needed by the masses is infinitesimally small. . . . The first group of experts has so far presented but few if any valid proofs that high school algebra deserves an obligatory position in the curriculum. The members of the second group are in still worse position. Their strongest argument is founded on falacious reasoning. They generally start from the right premise, "People use very little algebra," but arrive at the wrong conclusion, "Therefore they have very little need of it."—LIGDA: *The Teaching of Elementary Algebra* (1925).

Modern mathematics stands on a foundation of applied mathematics; without mathematics the earth could not support its present population. . . . The average man takes no direct part in these developments, but it is not fitting that he should live as a mere parasite on the organization that keeps him alive. . . . A public must be created able to realize what science and mathematics are doing for the world, and to form some general conception of the means. The average man will not be more than a spectator of the world's material progress: we have suggested that he may as well be taught to be an intelligent spectator.—GODFREY and SIDDONS: *The Teaching of Elementary Mathematics* (1931).

All mathematical teachers should reflect carefully on the nature of mathematical reasoning, and should see that their pupils are made more and more conscious of what constitutes mathematical rigour. Mathematical reasoning is not, as commonly supposed, *deductive* reasoning; it is based upon initial analysis of the given, and, being analytical, is in essence *inductive*. The threads of the web once disentangled, synthesis begins, and the solution of the problem is set out in deductive dress. We arrange our arguments deductively in order that other people may easily follow up the chain to our final conclusion. —F. W. WESTAWAY: *Craftsmanship in the Teaching of Elementary Mathematics* (1931).

The world needs a certain number of mathematicians to do its work: and as the world is prepared to pay for this, a certain number of boys at school must be learning mathematics with a view

of their future livelihood. The utilitarian argument is a perfectly respectable argument: we have refrained from putting it in the forefront, not through doubt as to its propriety, but because it applied to a relatively small proportion of boys and to a much smaller proportion of girls. —GODFREY and SIDDONS (1931).

While the movement to relate geometry to other subjects has had some beneficial influence on the teaching of geometry, the great body of teachers is convinced, after considerable experimentation, that demonstrative geometry will not blend with other subjects to any great extent without losing its chief educational value.—J. SHIBLI (1932).

Consider the rigid Euclidean lockstep through which we all plodded at school. Until we had been through the treadmill a dozen times we did not begin to see where the next step was to go, or where the one behind it had come from. To most of us it was all a sort of penal servitude. It was supposed to teach us deductive reasoning in the classical Greek pattern. But did it? Our assent to the logic of the whole dreary punishment was obtained by what we suspected of being a low subterfuge. We were clubbed into a dazed submission by a rap on the head with something as soft as a bologna sausage but as deadly as a blackjack. And when we first attempted to do some of the original exercises on our own account we woke up to the fact that we had not only been blackjacked but thoroughly swindled as well. We had never paid half enough attention to the simple-looking assumptions from which everything followed so smoothly.—E. T. BELL: *The Search for Truth* (1934).

So important is mathematics in contemporary civilization that for appreciative values, if for nothing else, it would be an injustice to the learner to let him miss the instruction and the discipline of mathematical study unless he proves himself to be utterly hopeless on these higher emergent levels of learning.—W. C. BAGLEY: *Education and Emergent Man* (1934).

It should be realized that because you do not make use of certain mathematical facts, it does not follow that these facts cannot be made use of or that it would not be profitable for you to make use of them . . . the real complaint is not that mathematics is not of much use to the ordinary educated man, but that neither the content of mathematics nor the methods adopted in the schools as at present organized, are of any very great use.—N. KUPPUSWAMI AIYANGAR: *Teaching of Mathematics in the New Education* (1935).

Every educated man must have sufficient

knowledge of that kind of mathematics which will make him understand and appreciate the important and increasing part played by mathematics in the progress of civilization and of society. Every man who wishes to be called educated should have sufficiently entered into the spirit of mathematical studies as to show a sympathetic acquaintance with their nature and scope.—N. KUPPUSWAMI AIYANGAR (1935).

I stand, despite attacks by uninformed minor educators, for defense of the basic elements of all true education, than which none enter more directly into all phases of the national existence and of the daily lives of every one than do the English language and the sort of mathematics that I have expounded—the sort that has any right to be called mathematics—the intelligent dealing with the quantities with which the world works. I desire that schools reform, not by banishing study about quantities because it has been poorly done, but rather by improving the teaching regarding these things, and by avoiding misconceptions, such as that which I mentioned concerning the nature of algebra. I demand for our children better—rather than worse—preparation than we have had to face a world in which quantities arise on every hand, in every walk of life, to every active woman. Thus, and by no other means, may our sons and daughters build a greater and a safer society in this world of quantitative things!—E. R. HEDRICK: The Meaning of Mathematics, *Scientific Monthly*, October, 1935.

Logic is a statement in technical form of the conditions under which reasoning is rigorously demonstrative. If the object of general education is to train the mind for intelligent action, logic cannot be missing from it. Logic is a critical branch of the study of reasoning. It remains only to add a study which exemplifies reasoning in its clearest and most precise form. That study is, of course, mathematics, and of the mathematical studies chiefly those that use the type of exposition that Euclid employed. In such studies the pure operation of reason is made manifest. The subject matter depends on the universal and necessary processes of human thought. It is not affected by differences in taste, disposition, or prejudice. It refutes the common answer of students who, conformable to the temper of the times, wish to accept the principles and deny the conclusions. Correctness in thinking may be more directly and impressively taught through mathematics than in any other way.—R. M. HUTCHINS: *Higher Learning in America*, 1936.

The primary purposes of the teaching of mathematics should be to develop those powers of understanding and of analyzing relations of quantity and of space which are necessary to an insight into and control over our environment and to an appreciation of the progress of civilization in its various aspects, and to develop those habits of thought and action which will make these powers effective in the life of the individual.—National Committee Report on the Reorganization of Mathematics (1923).

It is questionable whether two chess players have a moral right to pursue their game while the neighbor's house burns down. If mathematical thinking is a pattern of true thinking, then may there not be a moral obligation resting on the mathematician to demonstrate this truth before the world so that the world may profit by its value, so that the vast potentials of mathematics may be applied to problems of crying human need as well as to those of science? —S. T. SANDERS: Editorial, *National Mathematics Magazine* (Feb. 1937).

Anyone who was subjected to elementary geometry when his infantile brain was as unripe as a green walnut will recall the protracted misery he endured. Through stupid exercises of cutting out cardboard squares, rectangles, and circles, and measuring them and weighing them, he struggled to placate his teacher by 'rediscovering' the idiotically simple rules for finding the areas for such things. As scissor-and-balance gymnastics these tortures may have been an excellent initiation to the mysteries of a school laboratory in physics. As an introduction to Mathematics, and in particular to geometry, they were silly, incompetent, immaterial and irrelevant.—E. T. BELL: *The Handmaiden of the Sciences* (1937).

Mathematics being in its last analysis a type of rigorous but normal thinking should be subject to mastery as a method of thinking by every mind not abnormal or subnormal in character, i.e., by any normally constituted intelligence.—S. T. SANDERS: Editorial, *National Mathematics Magazine* (May 1938).

I believe that it is a great mistake to cut down the mathematics in our elementary physics to a "minimum." A simple analytic statement or proof is actually easier for the average student to grasp than a verbal explanation of the same thing in those cases where words may be substitued for symbols. What are symbols for, if not to simplify logical deductions? Not only are symbolic statements more concise, but also more easily understood, because many of our college students do not get much from what they read. Their mother tongue is Greek to them when it deals with ideas and uses words not found in the pulp magazines.—HENRY A. PERKINS, in *American Physics Teacher*, (1938).

The student of mathematics should be taught the history, the origin and the development of that vast subject, as well as the general relation of mathematics to the culture and to the practical life of civilized men. He should know the great names associated with mathematics, particularly in the period of its origin and early development. He should know who were Euclid, Archimedes, Hipparchus Ptolemy, Boethius. He should know what each of these great men did to promote the development of mathematics and what was the contribution of each to its contents and to its importance. ... In other words, it is quite as important for the college student to know about mathematics as it is for him to know something of mathematics itself.—NICHOLAS MURRAY BUTLER (1939).

Unquestionably, mathematics has made possible the mastery of elements of both the material and social environment of man which has resulted in numerous benefits to every individual in civilized society. These benefits are both direct and indirect. The number of individuals benefiting directly by their own study of mathematics varies inversely with the extent and importance of the benefit. ... However, the benefits resulting indirectly from the work of the few specialists are numerous and extend to all classes.—MINNICK: *Teaching Mathematics in the Secondary Schools* (1939).

I should like to see elementary geometry, as at present taught in all but a few schools in the United States, pitched neck and crop out of education.—E. T. BELL (1940).

The all-embracing, common enterprise of mathematics and science is the study of an ordered universe with the aid of an ordered mind, undertaken both for its own sake and for the continuous improvement of human living.—WILLIAM BETZ: MATHEMATICS TEACHER, (Dec. 1940).

The effort to make mathematics a prominent feature of education implies the desire to keep secondary education on a somewhat high level, both as regards the ideals which inspire it and the standards of achievement that are expected. ... It is true that our attempts at universal education will bring into the schools many a

case of Johnnie Lowique and Winnie Barelypass, as well as delightful Huck Finns who 'take no stock in mathematics.' Unless the American temper changes completely there is little danger that such boys and girls will be dealt with harshly, while we continue our efforts to devise studies and activities really suitable for them. But they should not be allowed to set the general pattern for education, any more than their tastes should be allowed too much weight in determining the pleasures and diversions available for educated people. A survey of movies and radio might indicate, however, that such a surrender is being made. In concluding, one might say that the statement of Hogben that the history of mathematics is the mirror of civilization, suggests that its position in the schools may reveal something not only about our conception of education, but our national philosophy and ideals.—Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics (1940).

We still have not done our duty to the pupil unless we devote some time to discussing some of the typical errors in reasoning. ... such topics as *ad hominen reasoning, reasoning in a circle, faulty analogies, poor authorities, non-causal relations, assuming a converse, avoiding the question, appeals to prejudice,* and the *technique of propaganda.* ... We cannot expect the English, or Latin, or Chemistry teacher to include them in their courses; the geometry teacher must do the work.—J. A. NYBERG (1941).

Teachers of mathematics also face a peculiar opportunity. When they accept the purpose of a genuinely social education and work wholeheartedly toward the achievement of that purpose, they find in their subject an instrument of great educational utility. They can teach mathematics for direct application to a wide variety of practical problems and for indirect application to countless others. They can use it for developing habits of precise generalization. They can employ it to direct the careful thinking in social situations upon which a good society must be based. When they meet this opportunity intelligently, they find their subject expanding and growing more valuable with every practical educational use.—DEAN HAROLD BENJAMIN (1941).

# Bobby Learns His Numerals

By ELIZABETH B. POTWINE
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THE following scenes from the history of number were written for presentation by students of junior, and of first year senior, high school age. They do not pretend to give a complete account of the development of the number system. They were given in a twenty minute assembly period, and, for dramatic purposes, we chose to represent the characters as children of the age of the actors and to make the action swift rather than the treatment of the subject complete.

Each scene was accompanied by a chart showing the numerals of the period. The charts were made in collaboration with the Art Department. They were of heavy white cardboard with numerals six inches high and $\frac{1}{4}$ to $\frac{1}{2}$ inches wide and were visible in all parts of the auditorium. They were placed on an easel by an attendant in costume who knew her cues and did the work swiftly and unobtrusively. A lantern and screen could be used instead of the charts. The use of charts has two advantages which we did not foresee. The charts were hung on exhibition in the hall after the presentation of the scenes, and the students enjoyed studying them at leisure. Also, they are surprisingly beautiful. The Egyptian and Chaldaean symbols have, perhaps, the most intrinsic beauty, but it is gratifying to see that a work of art can be made from our prosaic digits. A list of the charts follows this article.

The class elected a committee of students to study the proper costuming. We did not wish to go to any expense, so the costumes were assembled from material in the school property-room and from articles lent by our friends. Costumes are not necessary, but they add interest and color and make an artistic whole.

Some of the properties were made in the crafts shop. Among them were a painted headdress and collar for the Egyptian boy, the clay tablet of the Chaldaean, an abacus, and the bamboo book of Lilivati.

The entire project made an interesting correlation of mathematics with history, art, and crafts.

## CHARACTERS

BOBBY, *an American boy.*
BOBBY'S MOTHER.
AHMES, *an Egyptian boy of 2000 B.C.*
ABR-AM, a *Chaldaean boy of about 2000B.C.*
THEON, *a Greek of the classical period,*
GAIUS ⎱ *two Roman boys of the Augustan*
LUCIUS ⎰ *age.*
LILAVATI, *a Hindu girl.*
AL-KHOWAZMI ⎱ *two Arabian boys.*
OMAR-KHAYAM ⎰
LEONARDO OF PISA, *an Italian boy.*

*With the exception of Bobby's mother, the characters are represented as children about twelve years old. Lucius is younger.*

*The curtain rises showing a boy about twelve years old, dressed in pajamas and dressing-gown, working over his arithmetic. His table is strewn with books and papers.*

VOICE FROM OFF STAGE. Sleepy-head, sle-e-py-head! Time to put away your books!

BOBBY. Oh, mummie, just let me finish this example! I've got it almost done!

VOICE. Well, ten minutes more. But look out or you'll dream about figures.

BOBBY. I wish I knew who invented all this old stuff anyway. Wish I'd lived a thousand years ago before there were any fractions to plague me!

(*Bobby works on, growing drowsier and drowsier. His head drops on his forearm and he is fast asleep.*

*The curtain falls for a few seconds. When it rises, the screen on which the numerals are to be projected, or an easel with charts and attendant, is on the stage.*

*A boy in the Egyptian costume of 2000 B.C. enters.*)

BOBBY. Who are you?

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AHMES. I'm Ahmes. I lived in Egypt more than three thousand years ago. I heard you say that fractions bother you, so I came to sympathize with you.

BOBBY. What, did you have fractions?

AHMES. Oh yes, and problems and equations. Hard ones, too.

BOBBY. What were they like?

AHMES. Our earliest numerals were on stone monuments and were like these hieroglyphics on a royal tomb which show the number of the king's flocks. (*Chart I. Points and reads the numbers*).

Each of these figures tells a story. (*Chart II*) The numbers up to ten were merely scratches, two straight lines for two, six for six (*points to them as he names them*) One hundred is a coil of rope such as our surveyors used in measuring the fields; a thousand is a lotus flower, we had so many of them. Ten-thousand a man's hand, pointing in amazement. You'd never guess one-hundred thousand! A tadpole, because the mud of the Nile swarmed with them after each flood!

BOBBY. Yes, I remember about the plague of frogs. But how did you write a large number?

AHMES. This is the number of our king's captives. (Points it out on Chart III and reads it.)

BOBBY. What about those fractions?

AHMES. They were a nuisance. We had to change them all to a common numerator, one. (*Points to* $\frac{1}{4}$) This is one-fourth.

BOBBY. We change them to a common denominator.

AHMES. They have always been hard. Our teachers made tables to help in reducing them, but they were still difficult.

BOBBY. You said you had to solve problems?

AHMES. Yes, practical ones, such as the manager of a plantation needs in distributing bread to his slaves and mechanics. We were great agriculturists. We built granaries to store our wheat and millet.

BOBBY. Like our grain elevators?

AHMES. Somewhat; and we had to figure their cubical contents.

BOBBY. Whew! Did you have formulas and equations to help?

AHMES. Here's one. (*Chart IV. Reads and points*) Heap, that's your x, its whole, its seventh, it makes 19. The legs walking forward mean addition.

BOBBY. I've seen that in an ancient papyrus. My teacher says it is the oldest written equation. We should write $x + x/7 = 19$. (*writes*)

AHMES. Here is a problem I have always liked. It is in that same papyrus. (*Chart V*) See the numbers mount. Houses, 7; Cats, 49; Mice, 343; Ears of Corn, 2401; Grains of Corn, 16807.

BOBBY. Why, that's like our riddle,.

Kits, cats, sacks and wives,

How many were going to St. Ives?

*Ahmes exit. A black-haired, curly-headed boy rushes in, holding out a clay tablet which he displays, saying:*

ABR-AM. I'm Abr-am from Ur of the Chaldees over between the Tigris and Euphrates rivers. See what big numbers I had to learn!

BOBBY. Tell about them.

ABR-AM. My people were traders and lawmakers. In their work they used numbers like these wedge shaped marks pressed in soft clay bricks with a stylus and baked. (*Chart VI*) The vertical wedges are units; the horizontal, tens. This is fourteen.

In ordinary business we counted by tens as you do, but I went to the temple school at Bel-Nippur, where we counted by sixties. My old tablets are still in the ruins there. Here is one. (*Chart VII*) This line says "one 60 plus seven 10s equals two 60's plus one 10."

BOBBY. Why did the priests use sixty as a unit?

ABR-AM. Because they were astronomers. Our ancestors were wandering tribes. They came to love the stars which guide them across the desert, and to worship them.

BOBBY. But what has sixty to do with that?

ABR-AM. Our astronomer-priests divided the circle into three hundred and sixty

equal parts, perhaps because the year has three hundred and sixty-five days. We don't know, it was all so long ago. And they divided the great circle of the heavens into twelve regions, (*Chart VIII*) the signs of the Zodiac. You remember 60 goes six times into 360, even as the radius goes six times around the circle. (*Points to figures of the inscribed hexagon and the six-pointed star*)

BOBBY. Is that how we get our circular measure of 360 degrees and 90 degrees in a right angle?

ABR-AM. Yes, and your signs of the Zodiac, and your days of twenty-four hours of sixty minutes of sixty seconds each. They all come from our wise men who studied the stars. (*Exit*)

BOBBY. I wonder who comes next.

*Enters a Greek boy, Theon.*

THEON. I should come, but I never learned to do much reckoning. We Athenians left most of the business of calculating sums to our slaves. Oh, we did a little addition on counting frames, but we never worked out a really good system of writing numbers (*Chart IX*). We first used the initial letters; pi stood for pente, five, mu for myriads. Later we used the letters of the alphabet in order with a system of primes. (*Reads from the chart*)

BOBBY. Then you never studied arithmetic?

THEON. But yes, the properties of numbers! Odd numbers, even numbers, prime numbers, numbers built up in triangles and squares. (*Chart X*) Our great Pythagoras thought the universe was built on number. He offered a sacrifice of a hundred oxen when one of his brotherhood discovered the irrational number, the diagonal of the square, root-two, (*Points to chart*)

BOBBY. Sounds sort of looney to a plain American boy. Tell me about that counting frame.

THEON. There are Gaius and Lucius on their way to school. They will show you. They like practical things.

*Enter two Roman boys, Gaius, a lad of fourteen, carries an abacus, a bag of count-ers, stylus and tablets. Lucius, a little boy, is speaking to him.*

LUCIUS. Oh Gaius, see me do my finger counting. (*Holds up his hand, makes the counting signs, and counts*) unus, duo, . . . decem.

GAIUS. Good, little one.

LUCIUS. But I can't do my sums.

GAIUS. Show me one.

LUCIUS. (*Pointing to Chart XI and reading*) 278 et 64. These are too large for finger counting.

GAIUS. On this abacus put 8 and 4 counters on the first line, 7 and 6 on the next, 2 on the third.

LUCIUS. Yes.

GAIUS. Take ten off the first and put one in place of them on the second column. Take . . .

BOBBY, interrupting. So that's why we say we "carry one!"

GAIUS, continuing. Take ten off the second and put one on the third. How many have you?

LUCIUS. 342.

GAIUS. Optime!

BOBBY. That is clever. But none of you can multiply and divide with your numerals. And you have so many of them! We use only nine little figures and a zero, yet we can write huge numbers with them and make long computations. (*Chart XII*). Where did we get these queer symbols (*points to a three and an eight*) with which we do these long examples compactly and quickly?

*Enter a girl of twelve years in Hindu costume.*

LILAVATI. I think I can answer that question.

BOBBY. You, a girl!

LILAVATI. Yes, my father, Bhaskara, was a scholar and a poet. He wrote all his knowledge in a book of verse for me. There is much about numbers in it, and algebra.

BOBBY. I didn't suppose girls had to study such things in your time.

LILAVATI. They didn't, but you see my father learned from the stars the day and hour propitious for my marriage. The

hour cup was floating on the stream and we were watching for the water to rise to mark the fortunate moment, when a pearl from my marriage headdress fell and closed the opening. The proper time passed unnoticed and I was destined never to marry. To console me, my father wrote a book in my name, Lilavati, the beautiful, that my name might be known to latest times.

BOBBY. I call that hard luck. But about the numbers?

LILAVATI. On the walls of a cave in India are strange marks, probably made by traders who stopped there for shelter over night. (*Chart XIII*) Many people think these are the earliest forms of the numbers you use.

BOBBY. Did all your writers use these figures?

LILAVATI. No, our mathematicians wrote in Sanskrit (*points to Sanskrit on chart*), and their problems were in poetry, often very fanciful. The numbers were written out as words. Sometimes they were named after colors.

BOBBY. What kind of problems?

LILAVATI. About pipes filling cisterns, and the number of bees in a swarm. (*Reads one from her book*) The square root of half the number of bees in a swarm, and also 8/9 of the whole, alighted on the jasmines; and a queen bee buzzed responsive to the hum of a male inclosed in a lotus-flower, into which he had been allured at night by its sweet odor. Tell me, O beautiful damsel, the number of the bees.

BOBBY. And they could solve problems like that with words!

LILAVATI. And could complete the square of a quadratic. They showed the world how to do that.

But long before my day these nine digits which common traders employed had come into use and their knowledge had been carried across the great mountains by strangers from the west. There are two of them now.

*Two Arabian lads enter, dressed in costumes of the period of the" Arabian Nights."*

AL-KHOWARIZMI. Our forefathers were fierce Arabian tribesmen. By the sword they carried our religion through the dread Khyber Pass and to the Pillars of Hercules. Our caliph, Haroun-al-Raschid, brought to his court at Bagdad wise men from India and sages from Alexandria. We studied the Euclid of the Greeks and the algebra of India. By the way, you know, of course, that algebra is an Arabic word?

BOBBY. Meaning?

OMAR-KHAYAM. The "crossing over," because we learned to simplify the process of solving equations by transposing terms across the equality sign.

BOBBY. Gee! And you used the nine Hindu numerals?

OMAR. Yes. Many people today call them the Arabic numerals. (*They turn to go*)

BOBBY. Wait. Won't you tell me your names?

AL-K. This is Omar-Khayam. He was a good poet, but a better algebraist, and I am Al-Khowarizmi.

*Leonardo of Pisa, commonly called Fibonnacci, an Italian boy of the twelfth century, enters. He carries a large leather-bound book.*

LEONARDO. You know the rest of the story, don't you?

BOBBY. Do I?

LEONARDO. About the Crusades?

BOBBY (*puzzled*). Yes.

LEONARDO. And the Crusaders returning with gifts, silks and gems, perfumes and spices for wives and sweethearts, so that commerce developed between the Orient and Europe?

BOBBY. Oh yes, and the growth of the rich city republics of Italy. We had that in history.

LEONARDO. And trade routes down the Rhine and the Hanseatic League?

BOBBY. Of course. I begin to see.

LEONARDO. Well, that is the route the Arabic numerals traveled. It was a good thing, too, for the increased needs of commerce and banking required a better number system than the Roman and other ancient methods of notation.

BOBBY. What did you have to do about its introduction?

LEONARDO. My father was an official of Pisa. You would call him a consul, looking after the interests of our merchants in a seaport of North Africa. I used to play around the counting house and the harbor. I talked with the sailors. I saw the caravans come in with bales of merchandise and picked up some of the language of the swarthy Arab traders. They had a quick way of reckoning. When I grew up and had returned to Italy, I wrote about their numbers in my book on arithmetic.

BOBBY. Then you were the first European to use them?

LEONARDO. No, not exactly. One of the popes, Sylvester II, as a young man learned about them in Spain where there were Saracen universities. But he used them only to number the columns on his abacus. He did not reckon with them.

BOBBY. Weren't your countrymen awfully glad to learn about the new reckoning?

LEONARDO. No, they were very slow about taking it up. Some universities even passed laws forbidding the use of the Arabic numerals. The methods were long and clumsy at first.

(*Shows Chart XV and with pointer explains process of Gelosia Multiplication*)

(*Chart XVI*) The arrangement of this longer example shows why the method was called the Gelosia or "grating" method.

(*Chart XVII*) In division many figures used in the process were erased or scratched out.

(*Chart XVIII*). The work in long division was often arranged in a form which suggested a galley, a kind of ship propelled by oars. This example, taken from the first printed arithmetic, illustrates this so-called galley method of division. Notice that the boy who worked the problem, decorated it as you sometimes ornament your papers.

BOBBY. My! I guess arithmetic always has been hard.

LEONARDO. Just as hard for the human race as for you. But we have learned it.

*Exit Leonardo. Curtain, during which time charts or screen are removed.*

*Curtain rises showing Bobby at his desk, sleeping with head on his arm. Enters Mother. She shakes him gently to arouse him and picks up the scattered books and papers,*

MOTHER. Bobby, Bobby, wake up, dear.

BOBBY (*triumphantly*). Oh Mummie, I got that last example!

*Curtain.*

## CHARTS

I. Inscriptions from a royal tomb in Egypt with hieroglyphics, showing numbers of the king's flocks and herds. From Karpinski's History of Arithmetic. Rand McNally Co.

II. Egyptian hieroglyphs for 1, 2, 4, 10, 100, 1,000, 10,000, 100,000, ¼.

III. 212,346 in hieroglyphics.

IV. Two equations in hieroglyphics. a) Problem 33. Plate 55. b) Problem 27, Plate 47 from Rhind Papyrus, ed. by Chace, Bull, Manning.

V. Problem 79, Plate X, from Rhind Papyrus (The geometric progression).

VI. Chaldaean cuneiform numerals. Tablet from Bel-Nippur showing decimal and sexagesimal equivalents from Hilprecht, reproduced in a German history of mathematics. Sigmund Gunther, Sammlung Schubert XVIII, Leipzig.

VII. The Zodiac, signs and symbols arranged in a circle, in black and gold. The inscribed regular hexagon. The six pointed star.

VIII. Greek numerals. Capital initial letters. Later primed letters used in numbers, Triangular numbers of the series 1, 3, 6. 10 in geometric form. Square and its diagonal to illustrate root-2.

IX. Roman numerals for 1, 2, 4, 5, 10, 50, 100, 1,000. 278 plus 64 in Roman numerals.

X. The nine Arabic numerals and zero. A computation in these numerals.

XI. Copy, in color, of inscriptions in cave at Nana Ghat. Detail from these inscriptions. Sanskrit numerals 1 . . . 4. From D. E. Smith, History of Mathematics. Vol. 1.

XII. Development of the figure 8. From Fink, Brief History of Mathematics. Tr. W. W. Beman and D. E. Smith, Open Court.

XIII. A short example showing Gelosia multiplication.

XIV. A longer example showing Gelosia multiplication.

XV. An example showing scratch method of division. XIV, XV, XVI are from the Treviso Arithmetic, reproduced in D. E. Smith, History of Mathematics.

## A Mathematics Class Takes Music Lessons

*By* Lee J. Cronbach

*Fresno High School, Fresno, Calif.*

A Piano in a prominent place in the front of the geometry classroom rather astonished several classes at Fresno High School one day this spring. Their amazement grew as they perceived a table loaded with tuning forks, a sonometer, and a violin.

"What does mathematics have to do with music?" rose from them as a chorus when class started, and faded out only after several days of study opened their eyes to the fact, usually underemphasized in high school mathematics, that many persons far removed from engineering and science depend on mathematical principles to explain their work.

Proportion the students had heard of, and knew its use. But that they were applying it when they took a violin lesson after school came as a foreign and unbelievable idea. That they will appreciate the power of number science more, especially those among them who will take no later math or physics course, is impressively borne out by their comments, as the unit progressed, to the effect that they never knew mathematics stretched its influence so far.

Ratios present in simple rhythm patterns provided an easy opening for the bout with skepticism. A metronome was used to demonstrate how, though the absolute lengths of notes change, the ratio of a half-note to a quarter-note must be constant.

From this, attention turned to the sonometer and tuning forks, the piano and violin, which were used to discuss the role of ratios in explaining pitch, frequency, octaves, overtones, and harmony.

To round out the unit, the spotlight picked up the decibel scale for measuring loudness and showed its relation, not only to music conducting, but also to noise abatement inquiries, auditorium acoustics, and distortion in broadcasting.

Pythagoras' studies of the triangle were relegated while the class saw him as a music student, formulating the fundamental law that tones sound well together when the ratio of their frequencies involves only small integers. This fact was verified with the sonometer, which also demonstrated that the frequency of vibration of a tone on a stringed instrument is inversely proportional to the length of the string.

Musical ideas new to the students were brought out also. How our musical scale is constructed, and how it differs from that which makes Chinese music "weird" were studied from a mathematical viewpoint. "International" tuning was considered, and the standardizations that make the physicist's $C$ vibrate 512 times, compared to the American musician's 524 and the French musician's 529.

Genuine musical problems, solved by mathematics, accompanied practice exercises in musical proportions. No matter what the key, the ratio of the frequencies of *do* and *mi* is 5/4. Using this, the class learned to transpose such melodies as *Yankee Doodle* (CCDECED . . .) to the key of $A$. To solve these problems, a table showing the frequency of each note on the piano keyboard was given each student. Following solution of each problem on musical proportions, the answer was "played" on the piano, giving vivid evi-

dence of the musical rightness of a mathematical solution.

While the main purposes of the unit were to increase appreciation of the power of proportion, and to provide drill in solving numerical examples, a topic intended as subordinate nearly stole the show.

Much of the work of the conductor, in the past, has been the task of interpreting for the musicians exactly what rendition of the rather indefinite symbols for tempo and volume indicated in the score sounds best. Much of this demand on his skill would be spared could the composer replace such notations as *pp* and *fff* with decibel ratios to quantitatively tell each player how to modulate his tone, and similar improvements in exactness of other notations could (theoretically) make all music sound as the composer wished. This, of course, would eliminate the work of the conductor as "interpreter."

Horror-stuck music students revolted at this suggested desecration of the conductor's function, and equally rapidly were answered by scientific-minded boys who could visualize a control-room engineer, watching dials connected to microphones before each player, doing more skillfully the traditional duties of the orchestra leader. The debate, continuing over several class hours, served to air the merits of "canned" music and to add highlights to every discussion of the use of the decibel scale, particularly in broadcasting.

For most of the students, plane geometry is a terminal course; therefore it seemed appropriate to locate this experiment in enrichment where it would do the most for the greatest number of students. It could be adapted easily to enrich the unit in proportion for any mathematics course.

Approximately ten days seems the proper time allowance. An objective test, which showed substantial gains in information, followed the lessons. There was no effort to develop permanent musical skills.

Musical training on the part of the teacher was found unneeded, but understanding of physics desirable. Valuable aid was obtained from John Mills' *A Fugue in Cycles and Bels* (van Nostrand, 1935). The University of Chicago has produced a sound film explaining the physics of sound simply enough to be a valuable teaching aid here.

Despite the short time devoted to the unit, one outcome appears certain: the students have learned that one does not have to be a scientist or engineer to come daily under the influence of mathematics. And should the day arrive when music "goes mechanical," and the mixer room replaces the podium, at least one group will be ready to understand the change and the part of mathematics in a fuller living.

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* Deceased.

# ◆ IN OTHER PERIODICALS ◆

*By* Nathan Lazar

*The Bronx High School of Science, New York City*

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October, 1942, Vol. 49, No. 8

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2. Richardson, Moses, "On the Teaching of Elementary Mathematics," pp. 498–505.
3. Rosenbaum, R. A., "A Note on Joint Variation," pp. 537–538.
4. Sard, Arthur, "A Theorem on Improper Integrals," pp. 536–537.
5. Sholander, M. C., "The Linear Graph," pp. 543–545.
6. Whyburn, G. T., "What is a Curve?", pp. 493–497.

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October, 1942, Vol. 42, No. 7

1. Beach, James W., "A Laboratory Approach to Intermediate Algebra," pp. 615–616.
2. Boston, Paul F., and Sands, Lester B., "Curriculum Reorganization and the Mathematics-Science Program," pp. 671–675.
3. Carnahan, Walter H., "Determining the Place of Mathematics in the Educational Program," pp. 630–635.
4. Fuller, Evelyn G., "The Correlation of Mathematics and Science in One Unit," pp. 665–668.
5. Hussey, Arthur B., "Integral Solutions of Harmonic Equations," pp. 677–679.
6. Nyberg, Joseph A., "Notes from a Mathematics Classroom," pp. 661–664.

November, 1942, Vol. 42, No. 8

1. Bateman, Richard, "Method of Construction of a Parabola," p. 718.
2. Dawson, Lester, "Mathematics Helps Our Aerial Defense," pp. 722–723.
3. Mansfield, Ralph, "The General Equation of the Conic," pp. 729–736.
4. Moore, Lillian, "Aviation Mathematics," pp. 753–757.
5. Neuteiter, Paul R., "Our Neglected System of Notation," pp. 766–770.
6. Nyberg, Joseph A., "Notes from a Mathematics Classroom," pp. 762–765.
7. Teller, James D., "Harmonizing Science and Mathematics by Commemorating November Anniversaries," pp. 737–752.

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THE EIGHTH SUMMER MEETING OF THE
NATIONAL COUNCIL OF TEACHERS
OF MATHEMATICS

*By* Edith Woolsey, *Acting Secretary*
*Sanford Junior High School, Minneapolis,*
*Minn.*

The Eighth Summer Meeting of the National
Council of Teachers of Mathematics was held
at Denver, Colorado, June 29 and 30, 1942,
with headquarters at the Argonaut Hotel.

The theme of the meeting was: Mathematics
in These Times. A total of eight sessions were
held, a brief outline of which follows:

Monday, June 29, 1942.

1. 1:45 P.M. Joint session with the Department
   of Secondary Teachers. Theme: The Second-
   ary School Today.
2. 3:15 P.M. General Session of the National
   Council.
3. 6:00 P.M. Chuck Wagon Dinner at the Cam-
   pus of the University of Denver.
   Tuesday, June 30, 1942.
4. 12:15 P.M. Discussion Luncheon at the Olin
   Hotel.
5. 2:00 P.M. Section 1. Arithmetic in These
   Times.
6. 2:00 P.M. Section 2. Junior High School
   Mathematics in These Times.
7. 2:00 P.M. Section 3. Senior High School
   Mathematics in These Times.
8. 4:00 P.M. Multisensory Aids. Demonstration.

(A detailed report of this program was printed
in THE MATHEMATICS TEACHER for May, pp.
222–223.

An unusually fine exhibit was on display at
East High School under the direction of H. W.
Charlesworth. Many practical applications of
mathematics were shown, as well as the his-
torical development of mathematical devices.
The students, who made the materials exhibited,
showed considerable ability along the line of art
as well as in science and mathematics. The ex-
hibit was primarily the work of students.

The Chuckwagon Dinner was planned as a
typical western dinner in the out-of-doors, but
rain interferred, so it was held in one of the club
houses on the campus. This dinner lived up to
the advance advertising which promised "Fun,
fellowship, and fine food."

It is of interest to note that the first summer
meeting of the National Council was held in
Denver in 1935. The enthusiasm, hospitality,
and ability of the Denver mathematics teachers
made that a success, and to this group goes much
of the credit for the continued and growing

value of our summer meetings.

The local arrangements committee was very
efficient and neglected nothing that could have
been done to make this meeting a success. The
members were:

Wendell I. Wolf, chairman.
Grace Kenehan, luncheon.
Ernest R. Bails, outing.
Valworth R. Plumb, publicity.
Alfhild M. Alenius, hospitality.
H. W. Charlesworth, exhibit.
Margaret H. Aylard, miscellaneous.

A total of 213 people registered. The names follow:

Arizona
Prescott—Abbie Lee Taylor
California
Berkeley—Edith Mossman*
Los Angeles—Louis Bloch
Adeline Newcomb*
Charles W. Roadman*
Redondo Beach—H. P. McCandless
Florence McKenzie
Sacramento—Gertrude Harvie
Upland—Wm. A. Hawkinson
Colorado
Boulder—Mrs. E. B. Danielson
Harl R. Douglass*
Charles A. Hutchinson*
Claribel Kendall*
Berniece Neil
Reva Robison
Portia O. Woodbury
Canon City—Harold S. Lawrence*
Colorado Springs—Eugene L. Anderson
Mabel S. Bateman*
Mrs. Ruth Lee*
Charles Sisam
Madeleine Strong
Nellie V. Thompson
Del Norte—Maud Gray
Denver—Alfhild Alenius*
Sister Agnes Ruth
Margaret Aylard*
Ernest Bails
Florence Barnard
Mrs. Mae Bertagnolli
Ruth H. Bigham
Mary L. Brownfield
Brother Camillus, F. S. C.
H. W. Charlesworth*
Zula Charlesworth
Katherine Compton*
Elbert Cummings
Ella Davis

* Indicates members.

Marie Dormann
Burnell G. Downs
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Lon Edwards
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Nederland—Mary C. Holder
Oak Creek—Inez Bagwell
Pueblo—Faye Breckenridge
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W. M. Heaton
Isabelle Tour
Mildred E. Woodside
Rocky Ford—Ethel Z. Hutchinson
Trinidad—Florence Kendall*
District of Columbia
Washington—Etheyl L. Smith*
Florida
Miami—Mrs. Mary L. Franklin
Illinois
Bloomington—Frances McKern
Downers Grove—Ralph Cressey
Rockford—Katherine Slade*
Indiana
Elkhart—Rex Harvey
Michigan City—Mary Wright Gill
New Castle—Alice Smith Painter
Forty Wayne—Mabel E. Thorne*
P. Chas. Tupin*
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Avoca—Myra M. West
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Cedar Rapids—Elsie Cornely
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Ottumwa—Alta Marie Yearous*
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Kansas
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Athol—Letha Schoeni*
Cherryvale—Helen M. Forbes*
Emporia—Fannie Mae Kitts
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  Polytechnic—J. Josephine Leamer
Nebraska
  Franklin—E. A. Ward
  Hastings—Mrs. Alice Mutchler
    Wilma Scott
  McCook—Everett J. Lowry
  Peru—Arthur L. Hill*
  Rockville—Helen E. Ogle
  Weeping Water—F. Eloise Pool*
Nevada
  Carson City—James E. Pierce
New Mexico
  Albuquerque—Dorothy Shinn
  Artesia—Mary Woods
North Dakota
  Dickinson—Ruth S. Smith
  Fargo—Amanda Hanson*
Ohio
  Athens—R. L. Morton*
  Attica—Lloyd Kaufman
  Canfield—Bess B. Rice
  Cincinnati—Marie Hunter
  Fremont—Anna M. Setzler*
  Newark—Verna Keller
    Ethel M. Wente
  Struthers—Rebecca J. Gough
  Youngstown—Reba M. Hadley*
  Zanesville—Donald F. Summers
Oklahoma
  Alva—Evelyn C. Hoch
  Bartlesville—Marie Meade
  Bessie—Katherine Schilberg
  Holdenville—J. W. Bell
  Oklahoma City—Velma Elam*
    Myrtle Williams
  Paul's Valley—Wilba Clark
  Pawhuskg—Edith M. Layton
Pennsylvania
  Harrisburg—Carl Nevin
South Dakota
  Mitchell—Orcella Leonard
  Onida—Mary Ellen Briscoe
  Rapid City—Florence Krieger*
  Watertown—Carol Kramer
Texas
  Amarillo—Miss Ola Boyles
    Leta Johnson
    Kate Wilson*
    Miss Mannie Wilson
  El Paso—Hazel Osborn
  Kilgore—Ruth Choice
  Pampa—Loraine Bruce*
  Sulphur Bluff—Irene M. Hill

Utah
  Ogden—Frances Irene Parker
Washington
  Seattle—Olga L. Morsen
  Wenatchee—Vivian M. Sterling
Wisconsin
  Milwaukee—Adele Leonhardy
  Racine—Mary A. Potter*
    Velva Shufelt
Wyoming
  Cheyenne—Alice Williamson
  Laramie—Glennie Bacon*
    Ottis Rechard*
    Florence Reid
    Charles R. Scarbrough*
  Wheatland—Jon Stanley Peters

*Summary*

| | |
|---|---:|
| Arizona | 1 |
| California | 8 |
| Colorado | 106 |
| Dist. of Columbia | 1 |
| Florida | 1 |
| Illinois | 3 |
| Indiana | 5 |
| Iowa | 7 |
| Kansas | 12 |
| Massachusetts | 2 |
| Michigan | 1 |
| Minnesota | 4 |
| Missouri | 5 |
| Montana | 2 |
| Nebraska | 7 |
| Nevada | 1 |
| New Mexico | 2 |
| North Dakota | 2 |
| Ohio | 10 |
| Oklahoma | 8 |
| Pennsylvania | 1 |
| South Dakota | 4 |
| Texas | 8 |
| Utah | 1 |
| Washington | 2 |
| Wisconsin | 3 |
| Wyoming | 6 |
| Total | 213 |

## High School Victory Corps

Every high school student in the United States will have the opportunity to take a definite place in the national war effort through a voluntary enrollment plan announced on Sept. 25, 1942 by Paul V. McNutt, Federal Security Administrator and Chairman of the War Manpower Commission.

Endorsed by the Army, Navy, and the Commerce Departments and the National Council of Chief State School Officers and based upon the recommendations of the U. S. Office of Education Wartime Commission chairmaned by John

W. Studebaker, U. S. Commissioner of Education, the plan developed by a Policy Committee urges the creation of a High School Victory Corps in each public and private secondary school throughout the country.

Capt. Eddie Rickenbacker, flying ace of World War I and aviation leader, has at the request of Administrator McNutt, accepted chairmanship of the Victory Corps National Policy Committee. Capt. Rickenbacker will be out of the country for a while but upon his return expects to give the program of the Victory Corps his active attention.

Simultaneous with announcement in Washington, State superintendents and commissioners of education in the 48 States are asked to call on school boards and school officials to launch the program locally as soon as possible. A High School Victory Corps Manual* setting forth purposes, objectives, and recommended methods of organization was released today by the U. S. Office of Education and sent to all superintendents of schools and high school principals in the Nation.

"A realistic appraisal of our need for trained manpower, both in the Armed Forces and in war production, makes it evident that the high school can't go on doing business as usual," declares the Policy Committee in its statement. "High school youth are impelled by patriotic considerations to point their training to preparation for war work, to tasks requiring skill of hand and strength of body, coupled with intelligence and devotion. The 28,000 high schools of the Nation with their 6,500,000 students must voluntarily set about adaptation of their curricula and of their organization with all possible speed to train youth to do their part in the victory effort."

Two aims of the High School Victory Corps are: first, immediate, accelerated and special training of youth for that war service they will be expected to perform after leaving school; second, active participation of youth while still in school in the community's war effort.

Objectives which will be pursued both inside and outside the classroom are: (1) guidance of youth into critical services and occupations; (2) wartime citizenship training to insure better understanding of the war, its meaning, progress and problems; (3) physical fitness; (4) voluntary military drill for selected boys; (5) competence in science and mathematics; (6) pre-flight training in aeronautics for those preparing for air service; (7) pre-induction training for critical occupations; (8) community service including training for essential civilian activities.

* Available from the Supt. of Documents, Washington, D. C. 15 cents per copy.

### Basis of Membership

Every student enrolled in high school will be eligible to join the General Membership of the High School Victory Corps. Students within about two years of completing high school are eligible for admission to any one of the five special service divisions. These Victory Corps divisions are: (1) Land Service, which calls for pre-induction training for all branches of the Army except the air; (2) Air Service; (3) Sea Service, which provides training for all branches of the Navy except the air; (4) Production Service, preparing for war industries and agriculture, (5) Community Service, preparing for medical, nursing, teaching, and numerous other professions, and for business and civic services.

Girls, as well as boys, are welcome to Victory Corps ranks. Girls will predominate in the Production and Community Service divisions to meet growing demands of war industries, agriculture, nursing, business, and teaching.

What the plan will mean to the individual student enrolled in the Victory Corps has been detailed in the manual prepared for school administrators and principals. "Any student enrolled in a secondary school, who, in the judgment of the principal, headmaster, or other designated authority, meets the following simple requirements may be enrolled as a general member of the Victory Corps."

In order to hold general membership:

1. The student should be participating in a school physical fitness program appropriate to his abilities and probable contribution to the war effort.

2. The student should be studying courses of immediate and future usefulness to the war effort.

3. The student should be participating in at least one wartime activity or service such as air warden, fire watcher, Red Cross work, farm aid, salvage work, care of small children of working mothers, etc.

Requirements for membership in the five service divisions are more extensive. Thus, if a boy is within about two years of completing high school, and can pass the physical and mental tests, membership in the Air Service Division will require that he engage in at least three of the following:

1. Study at least one year of high school physics and three years of high school mathematics.

2. Study pre-flight aeronautics.

3. Study auto-mechanics, radio, electricity, or vocational shop courses in servicing, maintenance or repair of aircraft.

4. Participate in a physical fitness program.

5. Take military drill.

Course requirements, physical fitness, and

drill programs have been planned in light of requests from the Armed Forces. Army manuals already prepared and now in press will be used for certain pre-induction courses. Others are in preparation. A manual prescribing a physical fitness program to fit high school youth for war demands has been drafted already and soon will be sent to all schools. Basic to the whole Victory Corps plan is the aim of cutting down the time now needed to train men and women after they have enrolled in the Armed Forces or in war industries.

Product of many weeks of work by educational experts and the National Policy Committee, the High School Victory Corps provides the blueprints for the policy adopted by the U. S. Office of Education Wartime Commission on July 22. The Commission, representing many phases of public and private education, declared: "Opportunity should be provided through the schools for all in-school young people to participate in organized war effort."

### Endorsed by Department Heads

Secretary of War Henry L. Stimson praised the plan saying, "The War Department is heartily in favor of the proposal that the high school youth of America be given an opportunity to enroll in a Nation-wide organization under the direction of school authorities. The Victory Corps, with its emphasis on a thorough mastery of fundamental subjects—physical training, special studies, and other activities that properly can be a part of any school's program—will enable the boys and girls to serve more usefully after graduation both in the war effort directly, and indirectly in other related pursuits."

"Because the High School Victory Corps emphasizes both basic education and technical-vocational specialization, the Navy Department," declared Secretary Frank Knox, "feels that it will be an organization of great value both to the youth concerned and to the Nation in this war emergency."

Secretary Jesse H. Jones of the Department of Commerce announced that "The Civil Aeronautics Administration is prepared to collaborate to the fullest extent with the Office of Education and the War and Navy Departments in the development of the new program."

Although national in scope the High School Victory Corps will be administered by State and local school authorities. Each Chief State School Officer has been invited to name a State Victory Corps director and a State advisory committee composed of educators and civic group representatives.

### Local Administration

City and county superintendents have been asked to appoint local Victory Corps directors and community advisory committees. If a high school is large it is recommended that the principal name a Victory Corps director for the high school and appoint teachers to act as counselors for each of the respective divisions. Parents Victory Corps members, and teachers all would have membership on the High School Victory Corps Council which would help develop policies and plans.

Each youth who enrolls will sign the following pledge:

"In making this application I pledge myself, if accepted for membership, to strive to be worthy of wearing the general insignia of the Victory Corps. I will efficiently perform any community war services within the limits of my ability and experience; and I will diligently seek to prepare myself for future service whether in the Armed Forces, in war production, or in essential civilian occupations.

"In evidence of my present qualifications for general membership in the Victory Corps I submit the following statement of my program of studies and of my extracurricular activities and community services related to the Nation's war effort."

"It is not intended," declares the Policy Committee statement, "that the High School Victory Corps will supersede any existing voluntary organizations."

Members of the High School Victory Corps may wear insignia indicating their membership in the Corps as a whole and with special devices showing membership in special divisions. Each member also will be entitled to wear a service cap. Both insignia and caps may be made by the pupils themselves in connection with their work in home economics or art classes or these articles may be purchased from regular distributors. Although "no elaborate uniforms are recommended," the Victory Corps cap may be worn on all occasions of public appearance. If staff is lacking in the high school to supervise various Victory Corps activities, such as training in vocational specialties and physical fitness, school officials are urged to comb their communities and secure help from competent citizens.

Members of the National Policy Committee which prepared the Victory Corps plans are:

#### War Department

Lt. Col. Harley B. West, War Department General Staff, G-3 Division.

Major Francis Parkman, Office, Director of Individual Training, Headquarters, Army Air Forces.

#### Navy Department

Joseph W. Barker, Special Assistant to the Secretary of the Navy.

Lt. Commander Malcolm P. Aldrich, represent-

ing the Office of the Assistant Secretary for Air, Navy Department.

*Department of Commerce*

Civil Aeronautics Administration

William A. M. Burden, Special Aviation Assistant to the Secretary of Commerce.

*U. S. Office of Education Wartime Commission*

Selma M. Borchardt, Washington representative of the American Federation of Teachers.

L. H. Dennis, Executive Secretary, American Vocational Association.

Paul E. Elicker, Executive Secretary, National Association of Secondary School Principals.

Willard E. Givens, Executive Secretary, National Education Association.

*Civilian Aviation*

Frank A. Tichenor, Chairman of the Aeronautical Advisory Council Department of Commerce, Publisher *Aero Digest*.

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### Bibliography for War Courses

The following bibliography is not intended to be exhaustive, but suggestive of the kind of thing that may be helpful to teachers of mathematics and science. It includes only the more elementary works. Some of them contain material that is not important; so teachers will have to read them critically to get the valuable parts. —Editor.

A Review of Arithmetic. Prepared by Zens L. Smith, The Institute of Military Studies, University of Chicago, 1942. Price, 25¢.

Betz, Wm. Basic Mathematics. Ginn, 1942. Price, $1.48.

Bradley, A. Day. Mathematics of Air and Marine Navigation. American Book Co., 1942. Price, $1.00.

Buchan, Borthwick and Wadden. Houghton, Mifflin Co., 1942.

C. A. A. Bulletin, No. 21. "Primary Ground Study Manual."

C.A.A. Bulletin, No. 22. "Digest of the Civil Air Regulations."

C.A.A. Bulletin No. 23.

C.A.A. Bulletin, No. 24. "Practical Air Navigation."

C.A.A. Bulletin, No. 25. "Meteorology for Pilots."

C.A.A. Bulletin, No. 26. "Aerodynamics for Pilots."

C.A.A. Bulletin, No. 28. "Pilot's Power Plant Manual."

C.A.A. Bulletin, No. 29. "Pilot's Radio Manual."

"Civil Pilot Training Manual." U. S. Dept. of Commerce Civil Aeronautics Administration. Washington: U. S. Govt. Printing Office.

Downer, Alvin E. Practical Mathematics of Aviation. Pitman, 1939. Price, $1.00.

Eames, James P. Introduction to Aviation. Chemical Publishing Co., 1941. Price, $3.50.

Elementary Physics for Pilot Trainees. U. S. War Dept Technical Manual TM 1-233. Washington: U. S. Govt. Printing Office, 1942.

Hart, W. W. Basic Mathematics. Heath, 1942. Price, $1.52.

High School Victory Corps.
Victory Corps Series. Pamphlet No. 1.
Federal Security Agency, U. S. Office of Education.

Hooper, A. A Mathematics Refresher. Holt, 1942. Price, $1.90.

Jordanoff, Assen. Flying and How to Do It. Grosset and Dunlap, 1940. Price, $1.00.

Kramer, Edna E. Mathematics Takes Wings. An Aviation Supplement to Secondary Mathematics. Books 1, 2 and 3 for the 9th, 10th and 11th years. Barrie and Edwin, 1942. Price, 25¢ postpaid, 21¢ prepaid per pamphlet for 20 to 49 copies, 18¢ prepaid per pamphlet for 50 to 99 copies, 15¢ prepaid per pamphlet for 100 or more copies.

Mathematics Essentials for the War Effort. Florida Program for Improvement of Schools, Bulletin No. 40, State Dept. of Education, Tallahassee, Fla., 1942. Price, 63¢.

Naidich, James. Mathematics for the Aviation Trades. McGraw-Hill, 1942. Price, $1.80.

Osteyee, George. Mathematics in Aviation. The Air Education Series. Macmillan, 1942. Price, 64¢.

Pope, Francis and Otis, Arthur. Elements of Aeronautics. World Book Co., 1941. Price, $3.40.

Pre-Flight Aeronautics.
A Guide for a Course in Secondary Schools. Prepared by High School Teachers' Aeronautics Class. Summer Session, 1942. State College of Washington, Pullman, Wash. Price, 40¢ postpaid.

Practical Air Navigation. Civil Aeronautics Bulletin No. 24.

Meteorology for Pilots. Civil Aeronautics Bulletin No. 25.

Rogers, Wm. W. and Welton, Paul L. Blue Print Reading at Work. Silver Burdett Company, 1942. Price, $1.28.

Science and Mathematics and the War.
A Progress Report by the Philadelphia Regional Committee on Science and Mathematics Teaching, Reuben T. Shaw, Chairman. Northeast High School. Free while they last.

Seventeenth Yearbook of The National Council of Teachers of Mathematics. "A Source Book of Mathematical Applications." The Bureau of Publications, Teachers College, 525 W. 120 St., New York, N. Y. Price $2.

Shields, Bert A. Principles of Flight. The Introductory Aviation Series. McGraw-Hill, 1942.

Some Military Applications of Elementary Mathematics.
   Prepared by the Dept. of Mathematics, U. S. Military Academy, West Point, N. Y. The Institute of Military Studies, University of Chicago, 1942. Price, 15¢.
Teaching Aids. Training Division
   Bureau of Naval Personnel
   Navy Department, Washington, D.C.
   Free on request.
Thomas, J. M. Elementary Mathematics in Artillery Fire. McGraw-Hill, 1942. Price, $2.50.
Worlling and Hill. Aircraft Mathematics. Macmillan, 1942.
War Times.
   Official Bulletin, School Committee on War Services, Vol. I., No. 8. Board of Education, Newark, N. J. Dept. of Library and Visual Aids.
Woods, Karl D. Technical Aerodynamics. McGraw-Hill, 1942.

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VOLUNTEER TEACHING OF MATHEMATICS IN THE STATE OF MINNESOTA FOR MEN OUT OF SCHOOL AND ABOUT TO ENTER THE ARMED FORCES

By WILLIAM L. HART, *University of Minnesota*

In Minneapolis, a self-appointed commmittee was organized in September, 1942, to foster free mathematical instruction for men about to enter the armed forces. The main focus of the committee was on the needs of the young men who enlist as aviation cadets in the Army and Navy and who then wait, as civilians, for many months in some cases, before being called on to start training. The financial conditions of school boards in Minnesota seemed to rule out the possibility of quick and continuing action by them which would accomplish the main aim which the committee had in mind. The committee decided to sponsor courses of various types to be taught by volunteer teachers. Courses were outlined and the plan was taken up with enthusiasm and pressed vigorously all over Minnesota by the Committee on Education of the Minnesota Civilian Defense Council. Volunteer teachers were obtained, and class room facilities were arranged through the cooperation of the administrators of the public schools. In answer to the first announcement of courses, approximately eight hundred men were enrolled in classes, mostly offered in the evenings, in over ninety communities. In about one hundred smaller communities, the few men asking for mathematical aid are being assisted through individual tutoring. The majority of the men who appeared for these classes are carrying full time day work and yet they are spending several nights per week in studying mathematics. The response to this offer of instruction in Minnesota clearly shows the intense desire of the young men of military age to obtain mathematical knowledge. They deserve to be taught all the mathematics which they can assimilate in the brief time at their disposal. I urge volunteer service bureaus and mathematicians to consider the possibility of actions similar to the Minnesota plan in any community where *accelerated free instruction* in mathematics for adults is not continuously available through the public schools.

The courses recommended under the Minnesota plan are as follows, where each course is scheduled for two hours per night, three nights per week, for six weeks. Students are encouraged to continue with the next higher course, in sequence, until they are called to service in the armed forces. The specified prerequisites were designed to be interpreted liberally; their general intention was to locate each student at a level where he could learn the content to the point of mastery.*

*Course A.* A refresher course in arithmetic, the elements of algebra, and intuitional geometry. *Prerequisite:* Two years or less of mathematics taken above grade eight: or, advanced algebra taken in the dim past.

*Course B.* A very elementary treatment of plane trigonometry, emphasizing the trigonometry of acute angles, applications to aeronautics, logarithmic computation and the slide rule, and vector problems, with incidental review of necessary algebra and geometry. *Prerequisite:* Fresh knowledge of advanced high school algebra, or Course A.

*Course C.* Review of plane trigonometry, with emphasis as specified for Course B: simplified treatment of spherical trigonometry: applications of plane and spherical trigonometry to problems of geography, plane and middle latitude sailing, and celestial navigation. *Prerequisite:* Plane trigonometry or Course B.

It should be pointed out that, in accordance with the viewpoints of many responsible educators as well as of spokesmen of the Army and Navy, the preceding plan for instruction emphasizes basic *mathematical content* and does not merely attempt an imitation of specialized courses which are given currently in curricula of the schools of the Army, Navy, or Civilian Aeronautics Authority.

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* Information about these courses and their administration in Minneapolis can be obtained from Miss Marie Kallio, Adult Education Department, Minneapolis Public Schools, City Hall, Minneapolis, Minnesota. Information about the details of the statewide phases of the program can be obtained from Professor Clifford C. Archer of the University of Minnesota, Co-chairman, Committee on Education, Civilian Defense Council for the State of Minnesota.

At Teachers College, Columbia University, during the second semester, three new courses intended to serve the needs of the emergency will be offered for teachers of mathematics: by Dr. J. R. Clark, Professionalized Subject-Matter Course for Experienced Teachers of Other School Subjects; by Mr. Mirick, Dr. Hausle and others, The Teaching of Mathematics Related to War Courses; by Dr. Shuster, Elementary Military Engineering and Air Navigation for Teachers of Mathematics and Science.

## THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
### Report of the Treasurer
EDWIN W. SCHREIBER, Macomb, Illinois
*For the Year, February 1, 1941 to January 31, 1942*

Balance on hand at beginning of year:

| | | |
|---|---|---|
| Union National Bank of Macomb..................... | $1081.57 | |
| Savings bank Deposit......................... | 3437.66 | |
| Commonwealth Edison Bond, 3½%, June 1, '68......... | 2075.77 | |
| | 6596.00 | |
| Net loss on Book Value of Bond.................... | 1.89 | $6593.11 |

Receipts for the year:

| | | |
|---|---|---|
| Mathematics Teacher, W. D. Reeve................... | $1560.67 | |
| Kazir Monograph, W. D. Reeve..................... | 40.45 | |
| Interest on Bond.................................. | 70.00 | |
| Interest on Savings............................... | 14.12 | 1685.24 |
| | | 8278.35 |

Expenditures for the year:

Atlantic City Meeting:

| | | | |
|---|---|---|---|
| Directors' expenses............................... | $ 400.00 | | |
| Program speakers................................. | 171.10 | | |
| Local Committee................................. | 76.48 | | |
| Printing Programs............................... | 50.50 | | |
| Incidentals..................................... | 31.40 | $ 729.48 | |
| Boston Meeting................................... | | 181.15 | |
| Bethlehem Meeting............................... | | 108.87 | |
| Directors' expenses............................... | | 331.00 | |
| President's Office................................. | | 150.00 | |
| Arithmetic Committee............................. | | 132.70 | |
| Chairman State Representatives, Salary.............. | | 600.00 | |
| State Representatives Committee..................... | | 150.00 | |
| Seventeenth-Yearbook Committee.................... | | 450.00 | |
| Multi-Sensory Aids Committee...................... | | 125.00 | |
| Secretary-Treasurer's Office: | | | |
| Ballot expense.................................. | $ 150.00 | | |
| Postage and Supplies............................ | 150.00 | | |
| Stationery...................................... | 47.00 | | |
| Incidental expense.............................. | 7.90 | | |
| Secretary Service............................... | 300.00 | 654.90 | |
| Contingencies.................................... | | 189.94 | $3803.04 |
| | | | 4475.31 |

### Statement of Assets in Treasurer's Office
January 31, 1942

| | | | |
|---|---|---|---|
| Commercial Bank Deposit........................... | $1580.15 | | |
| Savings Bank Deposit............................... | 821.28 | | |
| Commonwealth Edison Bond, 3½% June 1, '68.......... | 2073.88 | (1941) | (1940) |
| | $4475.31 | $6595.00 | $6271.60 |

(Signed) EDWIN W. SCHREIBER, *Treasurer*

The above report has been audited and found correct.

(Signed) W. S. SCHLAUCH, *Auditor*